IYGB GCE

Core Mathematics C3

Advanced

Practice Paper Z

Difficulty Rating: 3.9600/1.9608

Time: 2 hours

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

$$x = \ln\left(y^2+9\right)^{\frac{3}{2}}.$$

Show clearly that

$$\frac{dy}{dx} = \frac{y}{3} + \frac{3}{y}. \qquad \textbf{(6)}$$

Question 2

Prove the validity of the following trigonometric identity

$$\frac{\sin 2\varphi}{\sin\varphi} - \frac{\cos 2\varphi}{\cos\varphi} \equiv \sec\varphi. \qquad \textbf{(5)}$$

Question 3

The equation of a curve C is

$$y = \frac{x}{1+2\ln x}, \; x \in \mathbb{R}, \; x > 0 .$$

The curve has a single turning point at P.

a) Show that the coordinates of P are $\left(\sqrt{e}, \frac{1}{2}\sqrt{e}\right)$. **(6)**

b) Find the exact value of $\frac{d^2y}{dx^2}$ at P and hence determine its nature. **(4)**

Question 4

$$y = \arctan x\,, \;\; x \in \mathbb{R}\,.$$

a) By writing $y = \arctan x$ as $x = \tan y$, show that

$$\frac{dy}{dx} = \frac{1}{1+x^2}\,. \qquad \mathbf{(5)}$$

The curve C has equation

$$y = \arctan x - 4\ln\left(1+x^2\right) - 3x^2\,, \;\; x \in \mathbb{R}\,.$$

b) Show that the x coordinate of the stationary point of C is a root of the equation

$$6x^3 + 14x - 1 = 0\,. \qquad \mathbf{(5)}$$

c) Show that the above equation has a root α in the interval $(0,1)$. **(2)**

The iterative formula

$$x_{n+1} = \frac{1-6x_n^3}{14} \quad \text{with } x_0 = 0\,,$$

is used to find this root.

d) Find, correct to 6 decimal places, the value of x_1, x_2 and x_3. **(2)**

e) Hence write down the value of α, correct to 5 decimal places. **(1)**

Question 5

A scientist investigating the growth of a certain species of mushroom observes that this mushroom grows to a height of 41 mm in 5 hours.

He decides to model the height, H mm, t hours after the mushroom started forming, by the equation

$$H = k\left(1 - e^{-\frac{1}{12}t}\right), \quad t \geq 0$$

where k is a positive constant.

a) Show that $k = 120$, correct to three significant figures. **(2)**

The equation

$$H = 120\left(1 - e^{-\frac{1}{12}t}\right), \quad t \geq 0,$$

is to be used for the rest of this question.

b) Determine the value of t when $H = 90$, giving the answer in the form $a \ln 2$, where a is an integer. **(4)**

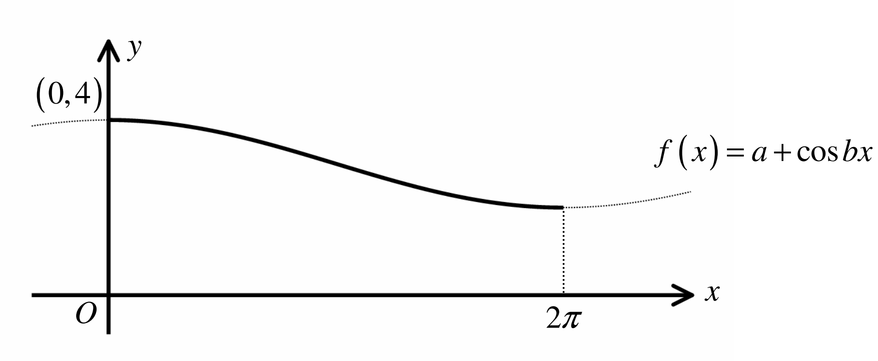
c) Show clearly that

$$\frac{dH}{dt} = 10 - \frac{1}{12}H.$$ **(3)**

d) Hence find the value of H when the height is the mushroom is growing at the rate of 7.5 mm per hour. **(2)**

e) State the maximum height of the mushroom according to this model. **(1)**

Question 6



The figure above shows the graph of

$$f(x) = a + \cos bx, \quad 0 \le x \le 2\pi,$$

where a and b are non zero constants.

The stationary points $(0,4)$ and $(2\pi, 2)$ are the endpoints of the graph.

a) State the range of $f(x)$ and hence find the value of a and b. **(5)**

b) Find an expression for $f^{-1}(x)$, the inverse function of $f(x)$. **(3)**

c) State the domain and range of $f^{-1}(x)$. **(2)**

d) Determine the gradient at the point on $f(x)$ with coordinates $\left(\frac{4}{3}\pi, \frac{5}{2}\right)$. **(2)**

e) State the gradient at the point on $f^{-1}(x)$ with coordinates $\left(\frac{5}{2}, \frac{4}{3}\pi\right)$. **(1)**

Question 7

It is given that

$$\sin P + \sin Q \equiv 2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right).$$

a) Prove the validity of the above trigonometric identity, by using the compound angle identities for $\sin(A+B)$ and $\sin(A-B)$. **(3)**

b) Hence, or otherwise, solve the trigonometric equation

$$\sin\theta - \sin 3\theta + \sin 5\theta = 0, \quad 0 \le \theta \le 180°.$$ **(6)**

Question 8

By considering the following sequence of transformations T_1, T_2 and T_3

$$\sqrt{x} \xrightarrow{T_1} \sqrt{x+1} \xrightarrow{T_2} \sqrt{|x|+1} \xrightarrow{T_3} \sqrt{|x+1|+1}$$

sketch the graph of $y = \sqrt{|x+1|+1}$. **(5)**

Indicate the coordinates of any intersections with the axes, and the coordinates of the cusp of the curve.
