IYGB GCE

Core Mathematics C3

Advanced

Practice Paper Y

Difficulty Rating: 3.9733/1.9737

Time: 2 hours

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

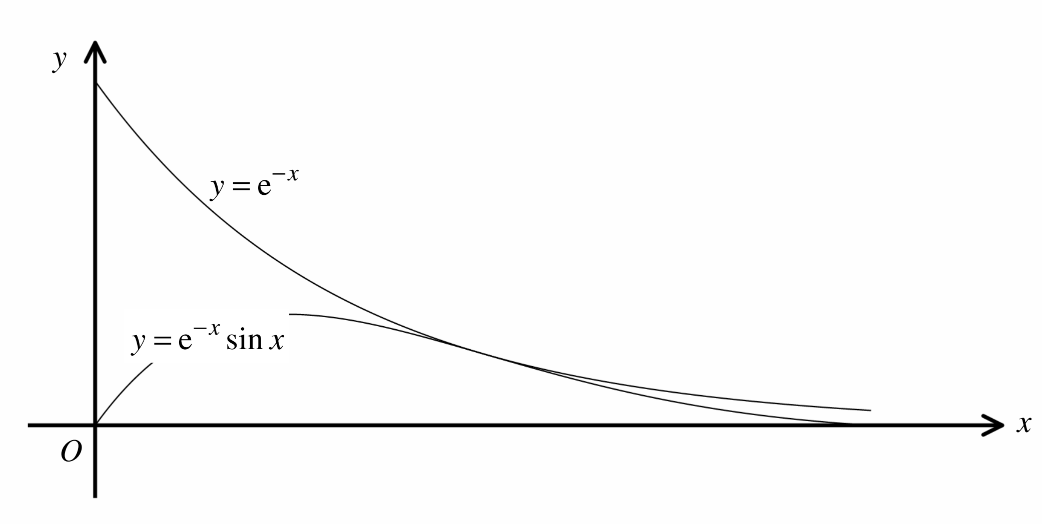
Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1



The figure above shows the graph of

$$y = e^{-x} \qquad \text{and} \qquad y = e^{-x}\sin x, \;\; 0 \le x \le \pi .$$

The curve with equation $y = e^{-x}\sin x$ has a local maximum at the point where $x = x_1$.

The curves touch each other at the point where $x = x_2$.

Show clearly that

$$x_2 - x_1 = \frac{\pi}{4} .$$ **(6)**

Question 2

The functions f and g are defined as

$$f(x) = 2\cos x + \sin x,\ x \in \mathbb{R}.$$

$$g(x) = \frac{5}{x^2+5},\ x \in \mathbb{R}.$$

a) Express $f(x)$ in the form $R\sin(x+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **(4)**

b) Determine the range of $gf(x)$, showing clearly all the relevant workings. **(4)**

Question 3

The mass, M grams, of a leaf t days after it was picked from a tree is given by

$$M = A\mathrm{e}^{-kt},\ t > 0$$

where A and k are positive constants.

When the leaf is picked its mass is 10 grams and 5 days later its mass is 5 grams.

a) Show clearly that

$$k = \tfrac{1}{5}\ln 2.$$ **(6)**

b) Find the value of t that satisfies the equation

$$\frac{dM}{dt} = \ln\left(\frac{\sqrt{2}}{2}\right).$$ **(6)**

Question 4

It is given that

$$\frac{d}{dx}(\tan 2x) = 2\sec^2 2x .$$

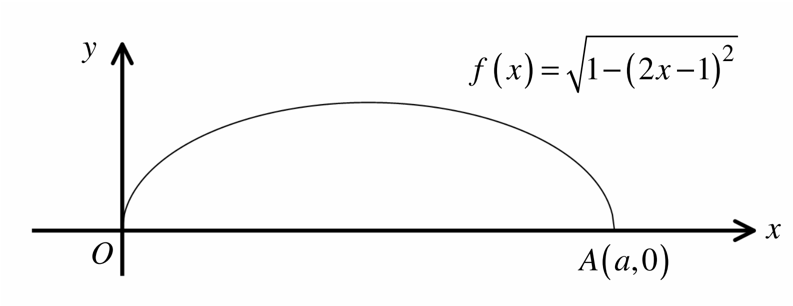
a) Prove the validity of the above result by considering the derivatives of $\sin 2x$ and $\cos 2x$. **(4)**

A curve has equation

$$y = 6x\tan 2x, \ x \in \mathbb{R} .$$

b) Show that the tangent to the curve at the point where $x = \frac{1}{8}\pi$ meets the y axis at the point with coordinates $\left(0, -\frac{3}{8}\pi^2\right)$. **(6)**

Question 5



The figure above shows the graph of the function

$$f(x) \equiv \sqrt{1-(2x-1)^2}\ ,\ x \in \mathbb{R},\ 0 \le x \le a\ .$$

a) Find the value of the constant a. **(2)**

b) State the range of $f(x)$. **(1)**

The function g is suitably defined by

$$g(x) = 2f\left(\tfrac{1}{2}x\right) - 2\ .$$

c) Sketch the graph of $g(x)$. **(4)**

d) State the domain and range of $g(x)$. **(1)**

Question 6

It is given that

$$\frac{2\cot\theta}{1+\cot^2\theta} \equiv \sin 2\theta\ .$$

a) Prove the validity of the above trigonometric identity. **(4)**

b) Use the above result to solve the trigonometric equation

$$4\cot^2\theta + 1 = 2\sin 2\theta\left(1+\cot^2\theta\right),\quad 0 \le \theta < 360°\ .$$ **(6)**

Question 7

The curves C_1 and C_2 have respective equations

$$y_1 = 3\arcsin(x-1) \quad \text{and} \quad y_1 = 2\arccos(x-1).$$

a) Sketch in the same set of axes the graph of C_1 and the graph of C_2.

The sketch must include the coordinates.

- … of any points where each of the graphs meet the coordinate axes.
- … of the endpoints of each of the graphs. **(5)**

b) Use a suitable iteration formula of the form

$$x_{n+1} = f(x_n) \text{ with } x_1 = 1.6,$$

to find an approximate value for the x coordinate of the point of intersection between the graph of C_1 and the graph of C_2. **(6)**

Question 8

A curve is defined by

$$f(x) = k\left(x^2 - 4x\right), \; x \in \mathbb{R},$$

where k is a positive constant

The equation $|f(x)| = 12$ has three distinct roots.

a) Determine the value of k. **(4)**

b) Find the three roots of the equation, giving the answers in exact surd form where appropriate. **(6)**
