C3, IYGB, PAPER X

1. a) $y = \dfrac{x^2 - 6x + 12}{4x - 11}$

$$\frac{dy}{dx} = \frac{(4x-11)(2x-6) - (x^2-6x+12)\times 4}{(4x-11)^2}$$

$$\frac{dy}{dx} = \frac{8x^2 - \cancel{24x} - 22x + 66 - 4x^2 + \cancel{24x} - 48}{(4x-11)^2}$$

$$\frac{dy}{dx} = \frac{4x^2 - 22x + 18}{(4x-11)^2}$$

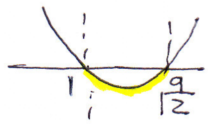
b) DECREASING $\Rightarrow \frac{dy}{dx} < 0$

$$\frac{4x^2-22x+18}{(4x-11)^2} < 0 \Rightarrow 4x^2 - 22x + 18 < 0$$

$$\Rightarrow 2x^2 - 11x + 9 < 0$$

$$\Rightarrow (2x-9)(x-1) < 0$$

C.V $<^{1}_{9/2}$



$1 < x < 9/2$

2. $\sin 2\theta = \cot\theta$

$\Rightarrow 2\sin\theta\cos\theta = \dfrac{\cos\theta}{\sin\theta}$

$\Rightarrow 2\sin^2\theta\cos\theta = \cos\theta$

$\Rightarrow 2\sin^2\theta\cos\theta - \cos\theta = 0$

$\Rightarrow \cos\theta(2\sin^2\theta - 1) = 0$ $\Big) \times(-1)$

$\Rightarrow \cos\theta(1 - 2\sin^2\theta) = 0$

$\Rightarrow \cos\theta\cos 2\theta = 0$

C3, IYGB, PAPER X

- $\cos\theta = 0$

$$\begin{cases} \theta = 90° \pm 360n \\ \theta = 270° \pm 360n \end{cases}$$

$n = 0, 1, 2, 3, \dots$

- $\cos 2\theta = 0$

$$\begin{cases} 2\theta = 90° \pm 360n \\ 2\theta = 270° \pm 360n \end{cases}$$

$$\begin{cases} \theta = 45° \pm 180n \\ \theta = 135° \pm 180n \end{cases}$$

$n = 0, 1, 2, 3, \dots$

$$\therefore \theta = 90°, 45°, 135°$$

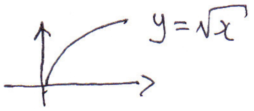
3. a) $f(x) = 1 + \sqrt{x} \quad x \in \mathbb{R}, x \geq 0$

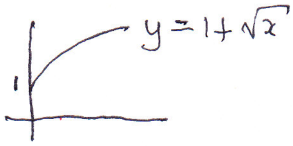
$$f(f(9)) = f(1+\sqrt{9}) = f(1+3) = f(4)$$
$$= 1 + \sqrt{4} = 1 + 2 = 3$$

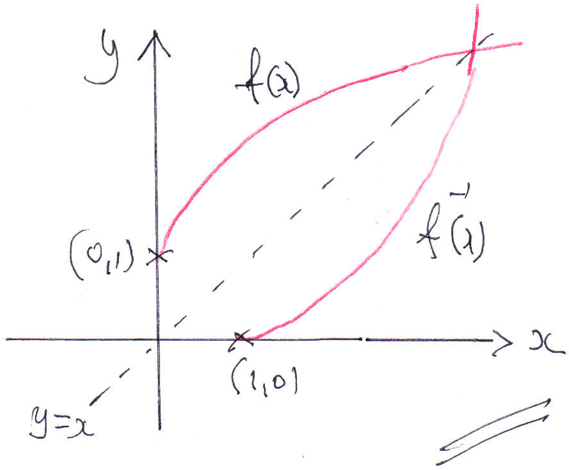
b) LET $y = 1 + \sqrt{x}$

$$y - 1 = \sqrt{x}$$
$$(y-1)^2 = x$$
$$\therefore f^{-1}(x) = (x-1)^2$$

c)







d) $f(x) = f^{-1}(x)$ is the same as $f(x) = x$

OR

$f^{-1}(x) = x$ (SEE GRAPH)

$$\Rightarrow (x-1)^2 = x$$
$$\Rightarrow x^2 - 2x + 1 = x$$
$$\Rightarrow x^2 - 3x + 1 = 0$$
$$\Rightarrow \left(x - \tfrac{3}{2}\right)^2 - \tfrac{9}{4} + 1 = 0$$
$$\Rightarrow \left(x - \tfrac{3}{2}\right)^2 = \tfrac{5}{4}$$
$$\Rightarrow x - \tfrac{3}{2} = \pm\tfrac{\sqrt{5}}{2}$$
$$\Rightarrow x = \tfrac{3}{2} \pm \tfrac{\sqrt{5}}{2}$$

BUT SOLUTION IS GREATER THAN 1

$$\therefore x = \tfrac{3}{2} + \tfrac{\sqrt{5}}{2}$$
$$x = \frac{3+\sqrt{5}}{2}$$

4. a) $\boxed{x = \ln(y^3 - 4y)}$

$$\Rightarrow \frac{dx}{dy} = \frac{1}{y^3 - 4y} \times (3y^2 - 4)$$
$$\Rightarrow \frac{dx}{dy} = \frac{3y^2 - 4}{y^3 - 4y}$$
$$\Rightarrow \frac{dy}{dx} = \frac{y^3 - 4y}{3y^2 - 4}$$
$$\Rightarrow 2 = \frac{y^3 - 4y}{3y^2 - 4}$$
$$\Rightarrow 6y^2 - 8 = y^3 - 4y$$
$$\Rightarrow 6y^2 - 8 = y(y^2 - 4)$$

$$\therefore y = \frac{6y^2 - 8}{y^2 - 4}$$

AS REQUIRED

b) $$y_{n+1} = \frac{6y_n^2 - 8}{y_n^2 - 4}$$

$P\left(\frac{11}{2}, \frac{13}{2}\right)$

$y_1 = \frac{13}{2} = 6.5$

$y_2 = 6.41830$

$y_3 = 6.43017$

$y_4 = 6.42841$

$y_5 = 6.42867$

$y_6 = 6.42863$

$y_7 = 6.42864$

$x \approx \ln\left(y_7^3 - 4y_7\right) \approx 5.480$

$\therefore P(5.480, 6.429)$ //

5. $$y = 4e^{2-x} - e^{4-2x}$$

$\frac{dy}{dx} = -4e^{2-x} + 2e^{4-2x}$

$\frac{d^2y}{dx^2} = 4e^{2-x} - 4e^{4-2x}$

- Now $\frac{dy}{dx} = 0$

$\Rightarrow -4e^{2-x} + 2e^{4-2x} = 0$

$\Rightarrow 2e^{4-2x} = 4e^{2-x}$

$\Rightarrow 2e^4 e^{-2x} = 4e^2 e^{-x}$

$\Rightarrow \frac{2e^4}{4e^2} = \frac{e^{-x}}{e^{-2x}}$

$\Rightarrow \frac{1}{2}e^2 = e^x$

$e^x = \frac{1}{2}e^2$

$x = \ln\left(\frac{1}{2}e^2\right)$

$x = \ln\frac{1}{2} + \ln e^2$

$x = -\ln 2 + 2\ln e$

$x = 2 - \ln 2$

TO GET y W/t CALCULATOR

OR

$$\boxed{e^{-x} = 2e^{-2}}$$

THUS $y = 4e^{2-x} - e^{4-2x}$

$y = 4e^{2}e^{-x} - e^{4}(e^{-x})^{2}$

$y = 4e^{2}(2e^{-2}) - e^{4}(4e^{-4})$

$y = 8 - 4$

$y = 4$

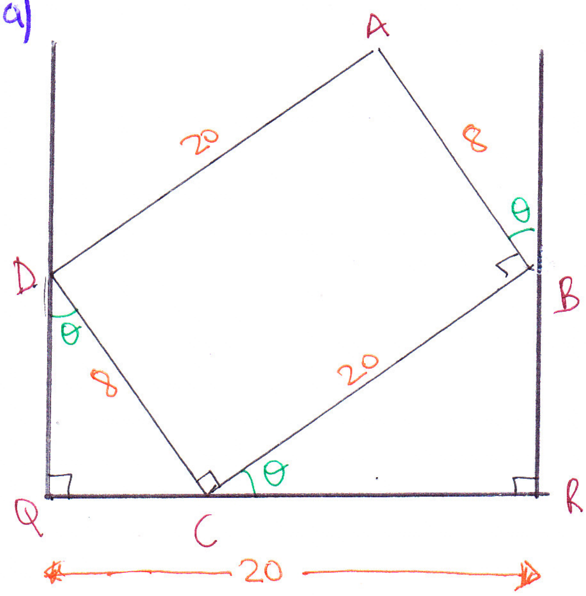
$\therefore (2-\ln 2, 4)$

FINALLY USING CALCULATOR (OR EXACT)

$$\left.\frac{d^2y}{dx^2}\right|_{\substack{x=2-\ln 2\\(e^{-x}=2e^{-2})}} = 4e^{2}(2e^{-2}) - 4e^{4}(2e^{-2})^{2}$$

$= 8 - 16 = -8 < 0 \quad \therefore$ MAX

6. a)



$|QC| + |CR| = 20$

$|DC|\sin\theta + |BC|\cos\theta = 20$

$8\sin\theta + 20\cos\theta = 20$

$2\sin\theta + 5\cos\theta = 5$

AS REQUIRED

b) $5\cos\theta + 2\sin\theta \equiv R\cos(\theta-\alpha)$
$\equiv R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$
$\equiv (R\cos\alpha)\cos\theta + (R\sin\alpha)\sin\theta$

$R\cos\alpha = 5$
$R\sin\alpha = 2$

SQUARE & ADD $R^2 = 5^2 + 2^2$
$R = \sqrt{29}$

DIVIDING $\tan\alpha = \frac{2}{5}$

$\alpha \simeq 21.80°$

$\therefore 5\cos\theta + 2\sin\theta \equiv \sqrt{29}\cos(\theta - 21.80°)$

c) $5\cos\theta + 2\sin\theta = 5$

$\Rightarrow \sqrt{29}\cos(\theta - 21.80°) = 5$

$\Rightarrow \cos(\theta - 21.80°) = \frac{5}{\sqrt{29}}$

$\arccos\left(\frac{5}{\sqrt{29}}\right) = 21.80°$

$\begin{cases} \theta - 21.80° = 21.80° \pm 360n \\ \theta - 21.80 = 338.20° \pm 360n \end{cases}$ $n = 0,1,2,3,\ldots$

$\begin{cases} \theta = 43.6 \pm 360n \\ \theta = 360 \pm 360n \end{cases}$

$\therefore \theta = 43.6°$

C3, IYGB, PAPER X

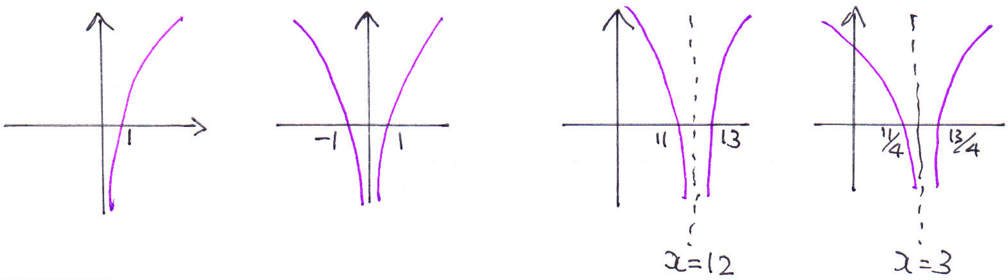
7. a) T_1 : THE PART OF THE CURVE FOR WHICH $x<0$ VANISHES (HERE IT DOES NOT MATTER)

THE PART OF THE CURVE FOR WHICH $x \geqslant 0$ STAYS, AND IS FURTHER REFLECTED IN THE y AXIS

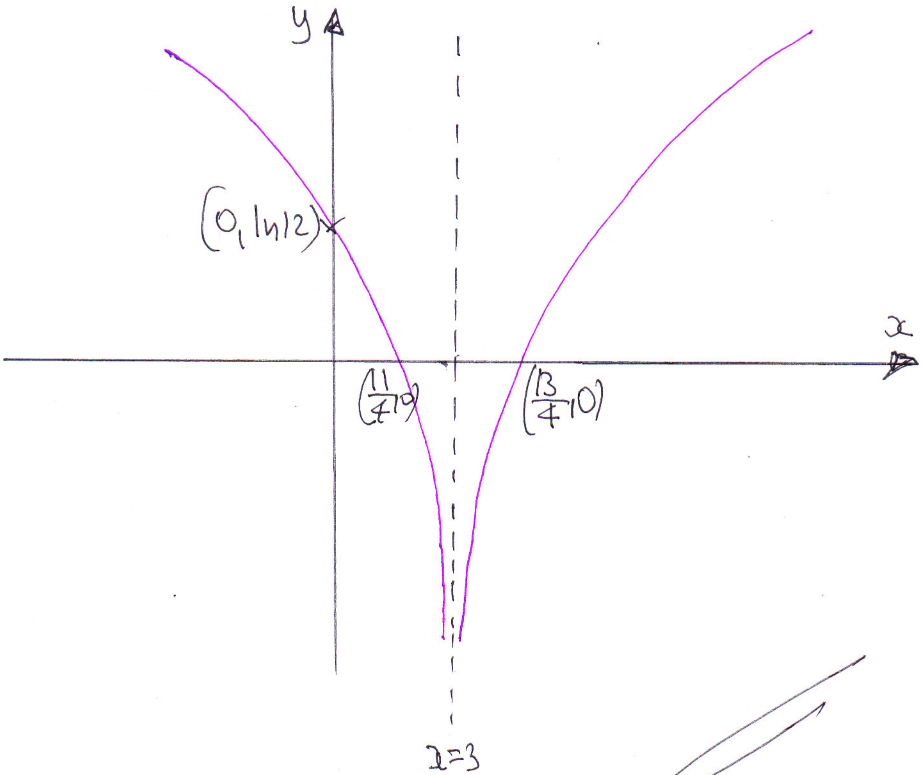
T_2 : TRANSLATION IN THE POSITIVE x DIRECTION BY 12 UNITS

T_3 : HORIZONTAL STRETCH BY SCALE FACTOR $\frac{1}{4}$

b)



$\boxed{y = \ln x}$ $\boxed{y = \ln|x|}$ $\boxed{y = \ln|x-12|}$ $\boxed{y = \ln|4x-12|}$



8. a) $y = x\sqrt{x+1} = x(x+1)^{\frac{1}{2}}$

$$\frac{dy}{dx} = 1 \times (x+1)^{\frac{1}{2}} + x \times \frac{1}{2}(x+1)^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = (x+1)^{\frac{1}{2}} + \frac{1}{2}x(x+1)^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}(x+1)^{-\frac{1}{2}}\left[2(x+1)^{1} + x\right]$$

$$\frac{dy}{dx} = \frac{1}{2}(x+1)^{-\frac{1}{2}}(2x+2+x)$$

$$\frac{dy}{dx} = \frac{1}{2}(3x+2)(x+1)^{-\frac{1}{2}}$$

AS REQUIRED

b) $f(x) = x\sqrt{x+1}\sin 2x$

BY PRODUCT RULE USING PART (a)

$$f'(x) = \frac{1}{2}(3x+2)(x+1)^{-\frac{1}{2}}\sin 2x + x\sqrt{x+1}(2\cos 2x)$$

$$f'(x) = \frac{(3x+2)\sin 2x}{2\sqrt{x+1}} + 2x\sqrt{x+1}\cos 2x$$

$$f'\left(\frac{\pi}{2}\right) = 0 + 2 \times \frac{\pi}{2}\sqrt{\frac{\pi}{2}+1} \times (-1)$$

$\sin\left(2 \times \frac{\pi}{2}\right) = 0$

$\cos\left(2 \times \frac{\pi}{2}\right) = -1$

$$f'\left(\frac{\pi}{2}\right) = -\pi\sqrt{\frac{\pi}{2}+1}$$

AS REQUIRED