Paper Reference(s)

## 6665/01

Edexcel GCE

## Core Mathematics C3

Advanced Subsidiary Level
Thursday 11 June 2009 - Afternoon
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Orange or Green)

Items included with question papers Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

## Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
1.


Figure 1
Figure 1 shows part of the curve with equation $y=-x^{3}+2 x^{2}+2$, which intersects the $x$-axis at the point $A$ where $x=\alpha$.

To find an approximation to $\alpha$, the iterative formula

$$
x_{n+1}=\frac{2}{\left(x_{n}\right)^{2}}+2
$$

is used.
(a) Taking $x_{0}=2.5$, find the values of $x_{1}, x_{2}, x_{3}$ and $x_{4}$.

Give your answers to 3 decimal places where appropriate.
(b) Show that $\alpha=2.359$ correct to 3 decimal places.
2. (a) Use the identity $\cos ^{2} \theta+\sin ^{2} \theta=1$ to prove that $\tan ^{2} \theta=\sec ^{2} \theta-1$.
(b) Solve, for $0 \leq \theta<360^{\circ}$, the equation

$$
2 \tan ^{2} \theta+4 \sec \theta+\sec ^{2} \theta=2
$$

3. Rabbits were introduced onto an island. The number of rabbits, $P$, $t$ years after they were introduced is modelled by the equation

$$
P=80 \mathrm{e}^{\frac{1}{5} t}, \quad t \in \mathbb{R}, \quad t \geq 0 .
$$

(a) Write down the number of rabbits that were introduced to the island.
(b) Find the number of years it would take for the number of rabbits to first exceed 1000.
(c) Find $\frac{\mathrm{d} P}{\mathrm{~d} t}$.
(d) Find $P$ when $\frac{\mathrm{d} P}{\mathrm{~d} t}=50$.
4. (i) Differentiate with respect to $x$
(a) $x^{2} \cos 3 x$,
(b) $\frac{\ln \left(x^{2}+1\right)}{x^{2}+1}$.
(ii) A curve $C$ has the equation

$$
y=\sqrt{ }(4 x+1), \quad x>-\frac{1}{4}, \quad y>0
$$

The point $P$ on the curve has $x$-coordinate 2. Find an equation of the tangent to $C$ at $P$ in the form $a x+b y+c=0$, where $a, b$ and $c$ are integers.
5.


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y=\mathrm{f}(x), x \in \mathbb{R}$.

The curve meets the coordinate axes at the points $A(0,1-k)$ and $B\left(\frac{1}{2} \ln k, 0\right)$, where $k$ is a constant and $k>1$, as shown in Figure 2.

On separate diagrams, sketch the curve with equation
(a) $y=|f(x)|$,
(b) $y=\mathrm{f}^{-1}(x)$.

Show on each sketch the coordinates, in terms of $k$, of each point at which the curve meets or cuts the axes.

Given that $\mathrm{f}(x)=\mathrm{e}^{2 x}-k$,
(c) state the range of f ,
(d) find $\mathrm{f}^{-1}(x)$,
(e) write down the domain of $\mathrm{f}^{-1}$.
6. (a) Use the identity $\cos (A+B)=\cos A \cos B-\sin A \sin B$, to show that

$$
\begin{equation*}
\cos 2 A=1-2 \sin ^{2} A \tag{2}
\end{equation*}
$$

The curves $C_{1}$ and $C_{2}$ have equations

$$
\begin{aligned}
& C_{1}: y=3 \sin 2 x \\
& C_{2}: y=4 \sin ^{2} x-2 \cos 2 x
\end{aligned}
$$

(b) Show that the $x$-coordinates of the points where $C_{1}$ and $C_{2}$ intersect satisfy the equation

$$
4 \cos 2 x+3 \sin 2 x=2
$$

(c) Express $4 \cos 2 x+3 \sin 2 x$ in the form $R \cos (2 x-\alpha)$, where $R>0$ and $0<\alpha<90^{\circ}$, giving the value of $\alpha$ to 2 decimal places.
(d) Hence find, for $0 \leq x<180^{\circ}$, all the solutions of

$$
4 \cos 2 x+3 \sin 2 x=2
$$

giving your answers to 1 decimal place.
7. The function f is defined by

$$
\mathrm{f}(x)=1-\frac{2}{(x+4)}+\frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R}, x \neq-4, x \neq 2 .
$$

(a) Show that $\mathrm{f}(x)=\frac{x-3}{x-2}$.

The function g is defined by

$$
\mathrm{g}(x)=\frac{\mathrm{e}^{x}-3}{\mathrm{e}^{x}-2}, \quad x \in \mathbb{R}, x \neq \ln 2
$$

(b) Differentiate $g(x)$ to show that $g^{\prime}(x)=\frac{\mathrm{e}^{x}}{\left(\mathrm{e}^{x}-2\right)^{2}}$.
(c) Find the exact values of $x$ for which $g^{\prime}(x)=1$
8. (a) Write down $\sin 2 x$ in terms of $\sin x$ and $\cos x$.
(b) Find, for $0<x<\pi$, all the solutions of the equation

$$
\operatorname{cosec} x-8 \cos x=0
$$

giving your answers to 2 decimal places.

## END

