



1.

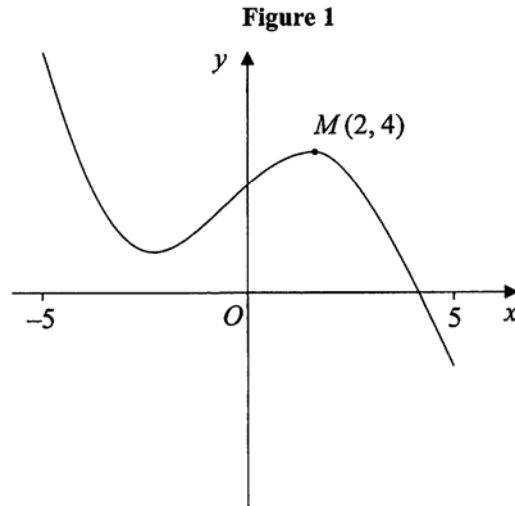


Figure 1 shows the graph of  $y = f(x)$ ,  $-5 \leq x \leq 5$ .  
The point  $M(2, 4)$  is the maximum turning point of the graph.

Sketch, on separate diagrams, the graphs of

(a)  $y = f(x) + 3$ ,

(2)

(b)  $y = |f(x)|$ ,

(2)

(c)  $y = f(|x|)$ .

(3)

Show on each graph the coordinates of any maximum turning points.

2. Express

$$\frac{2x^2 + 3x}{(2x+3)(x-2)} - \frac{6}{x^2 - x - 2}$$

as a single fraction in its simplest form.

(7)

3. The point  $P$  lies on the curve with equation  $y = \ln\left(\frac{1}{3}x\right)$ . The  $x$ -coordinate of  $P$  is 3.

Find an equation of the normal to the curve at the point  $P$  in the form  $y = ax + b$ , where  $a$  and  $b$  are constants.

(5)

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4. (a) Differentiate with respect to  $x$

(i)  $x^2e^{3x+2}$ , (4)

(ii)  $\frac{\cos(2x^3)}{3x}$ . (4)

(b) Given that  $x = 4 \sin(2y + 6)$ , find  $\frac{dy}{dx}$  in terms of  $x$ . (5)

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5.  $f(x) = 2x^3 - x - 4$ .

(a) Show that the equation  $f(x) = 0$  can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}. \quad (3)$$

The equation  $2x^3 - x - 4 = 0$  has a root between 1.35 and 1.4.

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{1}{2}\right)},$$

with  $x_0 = 1.35$ , to find, to 2 decimal places, the values of  $x_1$ ,  $x_2$  and  $x_3$ . (3)

The only real root of  $f(x) = 0$  is  $\alpha$ .

(c) By choosing a suitable interval, prove that  $\alpha = 1.392$ , to 3 decimal places. (3)

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6.

$$f(x) = 12 \cos x - 4 \sin x.$$

Given that  $f(x) = R \cos(x + \alpha)$ , where  $R \geq 0$  and  $0 \leq \alpha \leq 90^\circ$ ,

(a) find the value of  $R$  and the value of  $\alpha$ .

(4)

(b) Hence solve the equation

$$12 \cos x - 4 \sin x = 7$$

for  $0 \leq x < 360^\circ$ , giving your answers to one decimal place.

(5)

(c) (i) Write down the minimum value of  $12 \cos x - 4 \sin x$ .

(1)

(ii) Find, to 2 decimal places, the smallest positive value of  $x$  for which this minimum value occurs.

(2)

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7. (a) Show that

$$(i) \frac{\cos 2x}{\cos x + \sin x} \equiv \cos x - \sin x, \quad x \neq (n - \frac{1}{4})\pi, n \in \mathbb{Z},$$

(2)

$$(ii) \frac{1}{2}(\cos 2x - \sin 2x) \equiv \cos^2 x - \cos x \sin x - \frac{1}{2}.$$

(3)

(b) Hence, or otherwise, show that the equation

$$\cos \theta \left( \frac{\cos 2\theta}{\cos \theta + \sin \theta} \right) = \frac{1}{2}$$

can be written as

$$\sin 2\theta = \cos 2\theta.$$

(3)

(c) Solve, for  $0 \leq \theta < 2\pi$ ,

$$\sin 2\theta = \cos 2\theta,$$

giving your answers in terms of  $\pi$ .

(4)

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8. The functions  $f$  and  $g$  are defined by

$$f: x \rightarrow 2x + \ln 2, \quad x \in \mathbb{R},$$

$$g: x \rightarrow e^{2x}, \quad x \in \mathbb{R}.$$

(a) Prove that the composite function  $gf$  is

$$gf: x \rightarrow 4e^{4x}, \quad x \in \mathbb{R}. \quad (4)$$

(b) In the space provided on page 19, sketch the curve with equation  $y = gf(x)$ , and show the coordinates of the point where the curve cuts the  $y$ -axis. (1)

(c) Write down the range of  $gf$ . (1)

(d) Find the value of  $x$  for which  $\frac{d}{dx}[gf(x)] = 3$ , giving your answer to 3 significant figures. (4)

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END