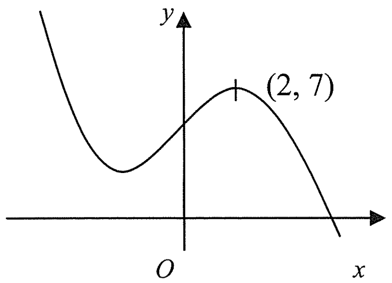
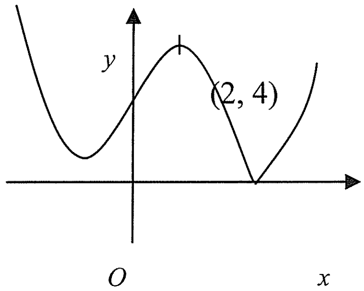
January 2006
6665 Core Mathematics C3
Mark Scheme

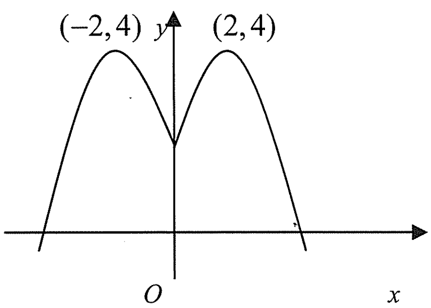
Question Number	Scheme		Marks
1.	(a)		
	Point	Shape unchanged	B1
			B1 (2)



Question Number	Scheme	Marks
	(b)	
	Shape	B1
	Point	B1 (2)



Question Number	Scheme	Marks
	(c)	
	Shape	B1
	(2, 4)	B1
	(−2,4)	B1 (3)
		Total 7 marks

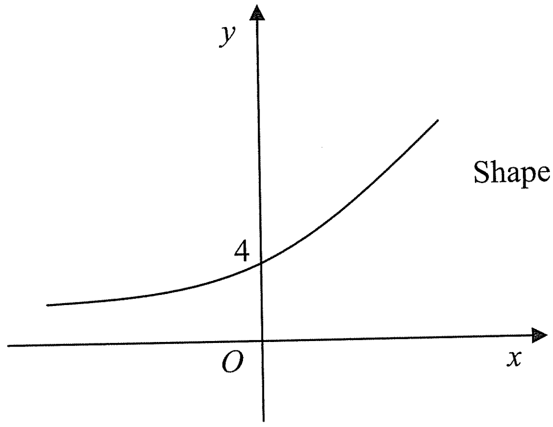


Question Number	Scheme	Marks
2.	$x^2-x-2=(x-2)(x+1)$ At any stage	B1
	$\dfrac{2x^2+3x}{(2x+3)(x-2)}=\dfrac{x(2x+3)}{(2x+3)(x-2)}=\dfrac{x}{x-2}$	B1
	$\dfrac{2x^2+3x}{(2x+3)(x-2)}-\dfrac{6}{x^2-x-2}=\dfrac{x(x+1)-6}{(x-2)(x+1)}$	M1
	$=\dfrac{x^2+x-6}{(x-2)(x+1)}$	A1
	$=\dfrac{(x+3)(x-2)}{(x-2)(x+1)}$	M1 A1
	$=\dfrac{x+3}{x+1}$	A1 (7)
		Total 7 marks
	Alternative method	
	$x^2-x-2=(x-2)(x+1)$ At any stage	B1
	$(2x+3)$ appearing as a factor of the numerator at any stage	B1
	$\dfrac{2x^2+3x}{(2x+3)(x-2)}-\dfrac{6}{(x-2)(x+1)}=\dfrac{(2x^2+3x)(x+1)-6(2x+3)}{(2x+3)(x-2)(x+1)}$	M1
	$=\dfrac{2x^3+5x^2-9x-18}{(2x+3)(x-2)(x+1)}$ can be implied	A1
	$=\dfrac{(x-2)(2x^2+9x+9)}{(2x+3)(x-2)(x+1)}$ or $\dfrac{(2x+3)(x^2+x-6)}{(2x+3)(x-2)(x+1)}$ or $\dfrac{(x+3)(2x^2-x-6)}{(2x+3)(x-2)(x+1)}$ Any one linear factor × quadratic	M1
	$=\dfrac{(2x+3)(x-2)(x+3)}{(2x+3)(x-2)(x+1)}$ Complete factors	A1
	$=\dfrac{x+3}{x+1}$	A1 (7)

Question Number	Scheme	Marks
3.	$\frac{dy}{dx}=\frac{1}{x}$ accept $\frac{3}{3x}$	M1 A1
	At $x=3$, $\frac{dy}{dx}=\frac{1}{3} \Rightarrow m'=-3$ Use of $mm'=-1$	M1
	$y-\ln 1=-3(x-3)$	M1
	$y=-3x+9$ Accept $y=9-3x$	A1 (5)
		Total 5 marks
	$\frac{dy}{dx}=\frac{1}{3x}$ leading to $y=-9x+27$ is a maximum of M1 A0 M1 M1 A0 =3/5	
4.	(a) (i) $\frac{d}{dx}\left(e^{3x+2}\right)=3e^{3x+2}$ $\left(\text{or } 3e^2e^{3x}\right)$ At any stage	B1
	$\frac{dy}{dx}=3x^2e^{3x+2}+2xe^{3x+2}$ Or equivalent	M1 A1+A1
		(4)
	(ii) $\frac{d}{dx}\left(\cos\left(2x^3\right)\right)=-6x^2\sin\left(2x^3\right)$ At any stage	M1 A1
	$\frac{dy}{dx}=\frac{-18x^3\sin\left(2x^3\right)-3\cos\left(2x^3\right)}{9x^2}$	M1 A1 (4)
	Alternatively using the product rule for second M1 A1	
	$y=(3x)^{-1}\cos\left(2x^3\right)$	
	$\frac{dy}{dx}=-3(3x)^{-2}\cos\left(2x^3\right)-6x^2(3x)^{-1}\sin\left(2x^3\right)$	
	Accept equivalent unsimplified forms	
	(b) $1=8\cos(2y+6)\frac{dy}{dx}$ or $\frac{dx}{dy}=8\cos(2y+6)$	M1
	$\frac{dy}{dx}=\frac{1}{8\cos(2y+6)}$	M1 A1
	$\frac{dy}{dx}=\frac{1}{8\cos\left(\arcsin\left(\frac{x}{4}\right)\right)}$ $\left(=(\pm)\frac{1}{2\sqrt{(16-x^2)}}\right)$	M1 A1 (5)
		Total 13 marks

Question Number	Scheme	Marks
5.	(a) $2x^2 - 1 - \frac{4}{x} = 0$ Dividing equation by x	M1
	$x^2 = \frac{1}{2} + \frac{4}{2x}$ Obtaining $x^2 = \ldots$	M1
	$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}$ * cso	A1 (3)
	(b) $x_1 = 1.41, x_2 = 1.39, x_3 = 1.39$	B1, B1, B1
	If answers given to more than 2 dp, penalise first time then accept awrt above.	(3)
	(c) Choosing $(1.3915, 1.3925)$ or a tighter interval	M1
	$f(1.3915) \approx -3 \times 10^{-3}, f(1.3925) \approx 7 \times 10^{-3}$ Both, awrt	A1
	Change of sign (and continuity) $\Rightarrow \alpha \in (1.3915, 1.3925)$	
	$\Rightarrow \alpha = 1.392$ to 3 decimal places * cso	A1 (3)
		Total 9 marks
6.	(a) $R\cos\alpha = 12, \quad R\sin\alpha = 4$	
	$R = \sqrt{(12^2 + 4^2)} = \sqrt{160}$ Accept if just written down, awrt 12.6	M1 A1
	$\tan\alpha = \frac{4}{12}, \Rightarrow \alpha \approx 18.43°$ awrt 18.4°	M1, A1 (4)
	(b) $\cos(x + \text{ their } \alpha) = \frac{7}{\text{their } R}$ (≈ 0.5534)	M1
	$x + \text{their } \alpha = 56.4°$ awrt 56°	A1
	$= \ldots, 303.6°$ 360° – their principal value	M1
	$x = 38.0°, 285.2°$ Ignore solutions out of range	A1, A1 (5)
	If answers given to more than 1 dp, penalise first time then accept awrt above.	
	(c)(i) minimum value is $-\sqrt{160}$ ft their R	B1ft
	(ii) $\cos(x + \text{ their } \alpha) = -1$	M1
	$x \approx 161.57°$ cao	A1 (3)
		Total 12 marks

Question Number	Scheme	Marks
7.	(a) (i) Use of $\cos 2x = \cos^2 x - \sin^2 x$ in an attempt to prove the identity.	M1
	$\dfrac{\cos 2x}{\cos x + \sin x} = \dfrac{\cos^2 x - \sin^2 x}{\cos x + \sin x} = \dfrac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x + \sin x} = \cos x - \sin x$ ✱ cso	A1 (2)
	(ii) Use of $\cos 2x = 2\cos^2 x - 1$ in an attempt to prove the identity.	M1
	Use of $\sin 2x = 2\sin x \cos x$ in an attempt to prove the identity.	M1
	$\frac{1}{2}(\cos 2x - \sin 2x) = \frac{1}{2}(2\cos^2 x - 1 - 2\sin x \cos x) = \cos^2 x - \cos x \sin x - \frac{1}{2}$ ✱ cso	A1 (3)
	(b) $\cos\theta(\cos\theta - \sin\theta) = \frac{1}{2}$ Using (a)(i)	M1
	$\cos^2\theta - \cos\theta\sin\theta - \frac{1}{2} = 0$	
	$\frac{1}{2}(\cos 2\theta - \sin 2\theta) = 0$ Using (a)(ii)	M1
	$\cos 2\theta = \sin 2\theta$ ✱	A1 (3)
	(c) $\tan 2\theta = 1$	M1
	$2\theta = \frac{\pi}{4}, \left(\frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}\right)$ any one correct value of 2θ	A1
	$\theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$ Obtaining at least 2 solutions in range	M1
	The 4 correct solutions	A1 (4)
	If decimals $(0.393, 1.963, 3.534, 5.105)$ or degrees $(22.5°, 112.5°, 202.5°, 292.5°)$ are given, but all 4 solutions are found, penalise one A mark only. Ignore solutions out of range.	**Total 12 marks**

Question Number	Scheme	Marks	
8.	(a) $\text{gf}(x) = e^{2(2x+\ln 2)}$	M1	
	$= e^{4x}e^{2\ln 2}$	M1	
	$= e^{4x}e^{\ln 4}$	M1	
	$= 4e^{4x}$ Give mark at this point, cso	A1	(4)
	$\left(\text{Hence gf} : x \mapsto 4e^{4x}, \quad x \in \mathbb{R}\right)$		
	(b)  Shape and point	B1	(1)
	(c) Range is $\mathbb{R}_{+}$ Accept $\text{gf}(x) > 0,\ y > 0$	B1	(1)
	(d) $\frac{d}{dx}\left[\text{gf}(x)\right] = 16e^{4x}$		
	$e^{4x} = \frac{3}{16}$	M1 A1	
	$4x = \ln\frac{3}{16}$	M1	
	$x \approx -0.418$	A1	(4)
		Total 10 marks	