

ASSIGNMENT TEST 5

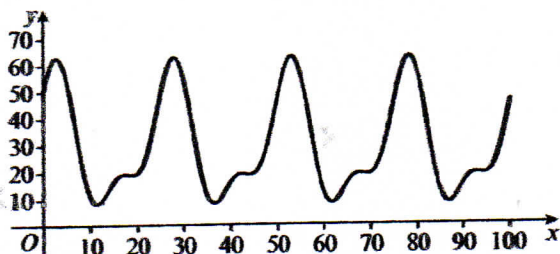
15 MARKS

20 MINUTES

1. a Use the substitution $t = \tan \frac{x}{2}$ to show that $\sec x + \tan x = \frac{1+t}{1-t}$,
 $x \neq (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$ (3)
- b Hence show that $\sec x + \tan x \equiv \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$ (3)

* identity proof
 set it out using
 LHS \equiv or RHS \equiv
 \equiv
 \equiv RHS \equiv
 \equiv LHS

2.



The diagram above shows the graph of $y = f(x)$ for the function

$$f(x) = 30 + 10 \sin \frac{x}{2} + 11 \sin \frac{x}{4} + 20 \cos \frac{x}{4},$$

$x \in [0, 100]$

a Show that

$$f'(x) = \frac{(t+1)(9t^3 - 49t^2 - 71t + 31)}{4(1+t^2)^2}$$

where $t = \tan \frac{x}{8}$ (6)

b Hence find the smallest exact multiple of π for which the graph has a stationary point. (2)

The function $kf(x)$ is used to model an electric pump which extracts L litres of water at time x seconds from a flooded mine shaft.

The maximum amount of water pumped is 300 litres.

c Suggest a suitable value of k . (1)