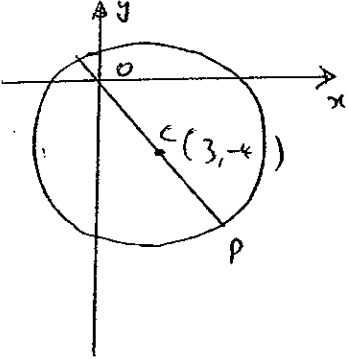


June 2005  
9801 AEA  
Mark Scheme

| Question Number | Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Marks                                                                  |
|-----------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------|
| 1               | <p> <math>(x-3)^2 + (y+4)^2 = 24 + 9 + 16 = 49</math><br/>           Curve is circle, centre <math>(3, -4)</math>, radius 7         </p>  <p> <math>OC = \sqrt{3^2 + 4^2} = 5</math> </p> <p>           Greatest length OP = <math>5 + r</math><br/>           (or least) <math>= \underline{12}</math> </p> <p>           Least length = <math>r - 5 = \underline{2}</math> </p> | <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>(6)</p> |

| Question Number | Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Marks                                                                          |
|-----------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------|
| 2<br>/          | $2 \sin \theta \cos \theta + \cos 2\theta + 1 = \sqrt{6} \cos \theta \quad (\text{use of } \sin 2\theta =)$ $2 \sin \theta \cos \theta + 2 \cos^2 \theta = \sqrt{6} \cos \theta \quad (\because 2 \cos^2 \theta \text{ needed})$ $\cos \theta (2 \sin \theta + 2 \cos \theta - \sqrt{6}) = 0$ $\cos \theta = 0 \Rightarrow \theta = \underline{\underline{\frac{\pi}{2}, \frac{3\pi}{2}}} \quad \begin{array}{l} (\text{Factor of } \cos \theta) \\ (\text{both}) \end{array}$ <p>or</p> $\sin \theta + \cos \theta = \frac{\sqrt{6}}{2}$ $\sqrt{2} \sin(\theta + \frac{\pi}{4}) = \frac{\sqrt{6}}{2} \quad \begin{array}{l} (\text{use of } \sin(\theta + \frac{\pi}{4}) \\ \text{or } \cos(\theta - \frac{\pi}{4})) \end{array}$ $\sin(\theta + \frac{\pi}{4}) = \frac{\sqrt{3}}{2}$ $\theta + \frac{\pi}{4} = \frac{\pi}{3} \text{ or } \frac{2\pi}{3} \quad (2 \text{ values})$ $\theta = \underline{\underline{\frac{\pi}{12}, \frac{5\pi}{12}}} \quad (\text{both})$ | <p>M1<br/>M1</p> <p>M1<br/>A1</p> <p>M1<br/>A1</p> <p>M1<br/>A1</p> <p>(8)</p> |

3

$$\frac{d}{dx}(u\sqrt{x}) = \sqrt{x} \frac{du}{dx} + \frac{1}{2\sqrt{x}} \cdot u \quad (\text{product rule}) \quad M1$$

$$\therefore \text{Eqn}^n \Rightarrow \sqrt{x} \frac{du}{dx} + \frac{1}{2\sqrt{x}} \cdot u = \frac{1}{2\sqrt{x}} \frac{du}{dx} \quad (\text{D.E.}) \quad A1$$

(all d's completed)

$$\therefore \frac{du}{dx} \left( \frac{1}{2\sqrt{x}} - \sqrt{x} \right) = \frac{1}{2\sqrt{x}} u$$

$$\therefore \frac{du}{dx} (1-2x) = u \quad (\text{simplification}) \quad M1, A1$$

$$\therefore \int \frac{du}{u} = \int \frac{dx}{1-2x} \quad (\text{sep}^n \text{ of variables}) \quad M1$$

$$\therefore \ln u = -\frac{1}{2} \ln(1-2x) \left[ + \frac{1}{2} \ln k \right] \quad (\text{int.}^n) \quad M1 (2 \text{ terms}), A1 (CAO)$$

(or  $\ln(2x-1)$ )

~~$$\text{Since } 0 < x < \frac{1}{2}, \ln u = \frac{1}{2} \ln(1-2x) + \frac{1}{2} \ln k$$~~

~~$$\frac{2}{k} = \frac{k}{1-2x}$$~~

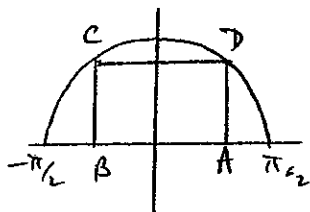
$$u=4, x=3/8 \Rightarrow 16 = \frac{k}{1-3/4} \Rightarrow k=4 \quad (\text{use of condition}) \quad M1$$

$$\therefore \underline{u = 2(1-2x)^{-1/2}}$$

A1

(9)

4 (a)



By symmetry, B is  $(-p, 0)$

$$\text{Area} = \underline{2p \cos p}$$

M1

A1

(2)

(b)

$$\frac{dA}{dp} = 2 \cos p - 2p \sin p$$

$$\frac{dA}{dp} = 0 \Rightarrow 1 = p \tan p, \text{ so when } p = \alpha, \underline{\alpha \tan \alpha = 1 \text{ o.e.}}$$

[This mark can be earned in (c)]

Let  $f(p) = p \tan p - 1$  (o.e.)

$$f(\pi/4) = \pi/4 - 1 < 0$$

(f at one end)  
( $f(\pi/4) < 0$ )

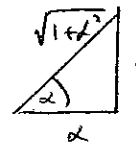
$$f(1) = \tan 1 - 1 = \tan 1 - \tan \pi/4$$

$$(1 > \pi/4) \therefore \tan 1 - \tan \pi/4 > 0 \text{ (Reason } f(1) > 0)$$

change of sign  $\therefore \underline{\pi/4 < \alpha < 1}$

(c)

Max. area =  $2\alpha \cos \alpha$ , with  $\tan \alpha = \frac{1}{\alpha} \Rightarrow$



$$= \frac{2\alpha^2}{\sqrt{1+\alpha^2}} \quad (= S)$$

M1 ( $\Delta$  or trig)

A1 c.s.o

(2)

(d)

$$\frac{dS}{d\alpha} = \frac{4\alpha \sqrt{1+\alpha^2} - 2\alpha^3 / \sqrt{1+\alpha^2}}{1+\alpha^2} = \frac{4\alpha + 2\alpha^3}{(1+\alpha^2)^{3/2}} > 0$$

$\therefore S$  is an increasing function as  $\alpha$  varies

$$S \text{ lies between } \frac{2(\frac{\pi}{4})^2}{\sqrt{1+(\frac{\pi}{4})^2}} = \frac{\pi^2}{2\sqrt{16+\pi^2}}$$

(subst.  $\alpha = 1$  and  $\alpha = \pi/4$ )

M1

$$\text{and } \frac{2(1)^2}{\sqrt{1+1}} = \sqrt{2}$$

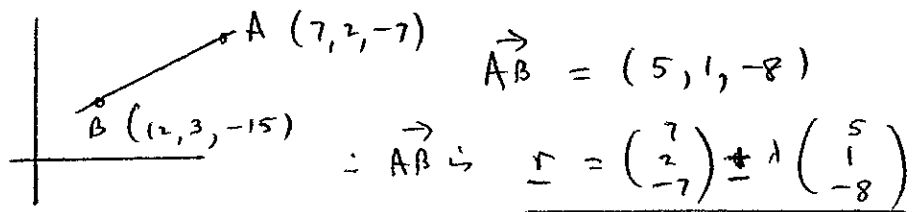
$$\text{i.e. } \underline{\frac{\pi^2}{2\sqrt{16+\pi^2}} < S < \sqrt{2}}$$

A1

(c.s.o)

(3)

5 (a)



$\vec{AB} = (5, 1, -8)$   
 $\therefore AB \hookrightarrow \underline{r} = \begin{pmatrix} 7 \\ 2 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 1 \\ -8 \end{pmatrix}$

M1, A1 (2)

(b)  $\mu = 4 \Rightarrow \vec{r} = (0, 0, 0) \therefore L_2$  passes through 0

B1 (1)

(c) If intersect,  $\left. \begin{aligned} 7 + 5\lambda &= -4 + \mu \\ 2 + \lambda &= 0 \\ -7 - 8\lambda &= 12 - 3\mu \end{aligned} \right\} \text{(Any 2 eqn's)}$

B1

Solving  $\Rightarrow \lambda = -2, \mu = 1$  (solving 2 eqn's)

M1

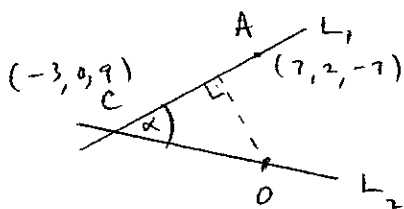
check in third equation  $(7 - 10 = -4 + 1$   
 or  $-7 + 16 = 12 - 3)$

$\therefore \vec{OC} = \begin{pmatrix} -3 \\ 0 \\ 9 \end{pmatrix}$

(check 3<sup>rd</sup> eqn + answer)

A1 (3)

(d)



$\vec{AC} = (-10, -2, 16)$  ( $\vec{AC}$ , allow  $\pm$ )  
 (or any vector along  $L_1$ )

M1

$\vec{AC} \cdot \vec{OC} = 30 + 0 + 144 = 174$

$= \sqrt{360} \cdot 3\sqrt{10} \cos \alpha$

$\therefore \cos \alpha = \frac{174}{6 \times 10 \times 3} = \frac{29}{30}$

(dep) M1, A1 (3)

(e) Shortest distance =  $|\vec{OC}| \sin \alpha = 3\sqrt{10} \cdot \sqrt{1 - (\frac{29}{30})^2}$

(complete method for M2)

$= 3\sqrt{10} \cdot \frac{\sqrt{59}}{3\sqrt{10}\sqrt{10}} = \sqrt{\frac{59}{10}}$  or

M2, A1 (3)

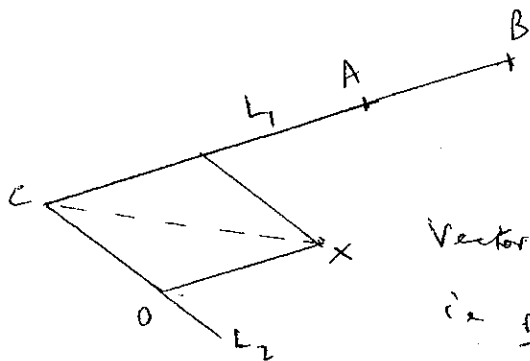
(f)  $|\vec{CO}| = 3\sqrt{10}$  ;  $|\vec{AB}| = \sqrt{25 + 1 + 64} = 3\sqrt{10}$

M1 (both lengths)

$\therefore |\vec{CO}| = |\vec{AB}|$

A1 (2)

(g)



$\vec{CX} = \vec{CO} + \vec{OX} = \vec{CO} + \vec{AB}$   
 $= \begin{pmatrix} 3 \\ 0 \\ -9 \end{pmatrix} + \begin{pmatrix} 5 \\ 1 \\ -8 \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \\ -17 \end{pmatrix}$  (dep)

M1

M1, A1

Vector eq<sup>n</sup> of bisector  $\vec{r} = \vec{OC} + t \vec{CX}$

M1 (dep)

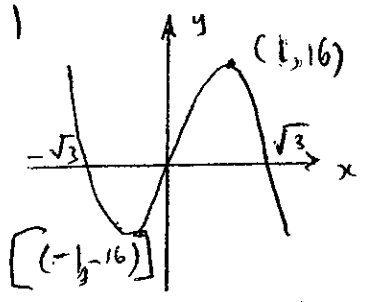
$\therefore \vec{r} = \begin{pmatrix} -3 \\ 0 \\ 9 \end{pmatrix} + t \begin{pmatrix} 8 \\ 1 \\ -17 \end{pmatrix}$

A1 (5)

6 (a)  $f = 0, x^2 = 12 \therefore R \text{ and } P \text{ are } (\pm\sqrt{12}, 0)$   
 $f' = 12 - 3x^2 \quad f' = 0, x^2 = 4; \text{ so } Q = (2, 16)$

B1, B1  
 M1, A1 (4)

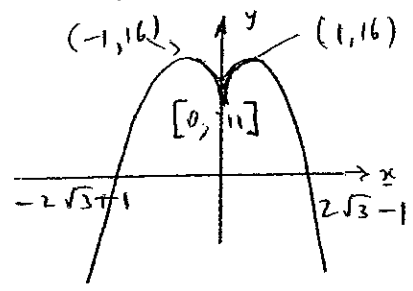
(b) (i)



shape, symmetry about 0  
 Any one of P', R', Q'  
 all correct

B1  
 M1  
 A1 ✓

(ii)



shape, symmetry about axis  
 $(2\sqrt{3}-1, 0)$  and  $(1, 16)$   
 & meets y-axis above origin  
 $(-2\sqrt{3}+1, 0)$  and  $(-1, 16)$

B1  
 B1  
 B1 (6)

(c)

Min. at  $(-2, -16) \Rightarrow (2, 0)$   
 $\therefore$  curve has "moved up" by 16  $\therefore w = 16$

M1, A1

$x=0 \Rightarrow f(-v) + w = 2 \times 16$  ( $\because S, U$  have same y-coordinates)

M1

$\therefore f(-v) = 16$   
 $-v(12 - v^2) = 16$

$v^3 - 12v - 16 = 0$  (correct cubic in v) A1

$(v+4)^2(v-4) = 0$  (finding a root) M1

$\therefore v = -2$  or  $v = 4$

Min. has moved from -2 to +ve value, so  $v > 0$

A1

$\therefore v = 4$

B1

Horizontal movement of 4 so T is (2, 0)

$S = (0, 32); T = (2, 0); U = (6, 32)$  so by symmetry

M1, A1

other point of intersection is  $5x = 8$

(9)

6 (c) Alternative solution. (algebraic)

$$\begin{aligned}\text{Let } g(x) &= f(x-v) + w \\ &= (x-v)(12 - (x-v)^2) + w \\ &= 12(x-v) - (x-v)^3 + w\end{aligned}$$

$$g'(x) = 12 - 3(x-v)^2 = 0 \text{ when } (x-v)^2 = 4 \\ x-v = \pm 2$$

Smaller of these values is the min.  $\therefore x = v - 2$  at min.

$$g(v-2) = -24 + 8 + w = 0 \quad \therefore \underline{w = 16}$$

(complete method for M1)

$$g(0) = -12v + v^3 + 16$$

$$g(v+2) = 24 - 8 + 16 = 32 \text{ (max. point)}$$

$$g(0) = g(v+2) \quad \therefore v^3 - 12v - 16 = 0 \quad (g(0) = g(v+2)) \text{ M1}$$
$$(v+2)^2(v-4) = 0 \quad (\text{correctable}) \text{ A1}$$

$$\therefore v = -2 \text{ or } v = 4 \quad (\text{a root}) \text{ M1}$$

$v = -2$  means min is at  $(-4, 0)$  - moved to the left, so not valid  $\therefore \underline{v = 4}$  A1

$$\therefore \underline{T \text{ is } (2, 0) \text{ i.e. } x = 2} \text{ B1}$$

$$g(x) = 12(x-4) - (x-4)^3 + 16$$

$$\Rightarrow x^3 - 12x^2 + 36x - 32 = 0$$

$$(x-2)^2(x-8) = 0$$

$$\therefore \underline{\text{other root is } x = 8} \text{ M1, A1}$$

(9)

|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                          |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------|
| 7 (a) | $x = \sec \theta ; \quad dx = \sec \theta \tan \theta \, d\theta$ $I = \int (\sec^2 \theta - 1)^{1/2} \sec \theta \tan \theta \, d\theta \quad (\sec^2 \theta - 1)$ $= \int \underline{\sec \theta \tan^2 \theta \, d\theta}$                                                                                                                                                                                                                                                                                                                        | M1<br>M1<br>A1 (3)                                       |
| (b)   | $J = \int \tan \theta (\sec \theta \tan \theta \, d\theta) \quad (\text{correct } u, v)$ $= \sec \theta \tan \theta - \int \sec \theta \cdot \sec^2 \theta \, d\theta$ $= \sec \theta \tan \theta - \int \sec \theta (1 + \tan^2 \theta) \, d\theta \quad (\text{split } \sec^3)$ $= \sec \theta \tan \theta - \int \sec \theta \, d\theta - J$ $\therefore 2J = \sec \theta \tan \theta - \ln  \sec \theta + \tan \theta  + C$ $\therefore J = \frac{1}{2} \left[ \sec \theta \tan \theta - \ln  \sec \theta + \tan \theta  \right] + \text{const}$ | M1<br>A1, A1<br>M1, A1<br>M1 (collect J's)<br>A1 cao (7) |
| (c)   | $K = \int_0^{\pi/4} \sin x \sqrt{2 \cos^2 x - 1} \, dx$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | M1                                                       |
| (*)   | $v = \sqrt{2} \cos x ; \quad dv = -\sqrt{2} \sin x \, dx \quad (v, dv \text{ both needed})$                                                                                                                                                                                                                                                                                                                                                                                                                                                          | M1                                                       |
|       | $\therefore K = -\frac{1}{\sqrt{2}} \int_{\sqrt{2}}^1 \frac{\sqrt{v^2 - 1}}{\sqrt{2}} \, dv \quad (\text{integrating})$                                                                                                                                                                                                                                                                                                                                                                                                                              | A1                                                       |
|       | $v = \sec \theta ; \quad dv = \sec \theta \tan \theta \, d\theta \quad (\sec \theta)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                | M1                                                       |
|       | $K = -\frac{1}{\sqrt{2}} \int_{\pi/4}^0 \sec \theta \tan^2 \theta \, d\theta \quad (\text{limits})$                                                                                                                                                                                                                                                                                                                                                                                                                                                  | M1                                                       |
|       | $= \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \left[ \sec \theta \tan \theta - \ln  \sec \theta + \tan \theta  \right]_0^{\pi/4}$                                                                                                                                                                                                                                                                                                                                                                                                                          | A1 (use of limits)                                       |
|       | $= \frac{1}{2\sqrt{2}} \left( \sqrt{2} - \ln(\sqrt{2} + 1) \right) \quad (\text{use of limits})$                                                                                                                                                                                                                                                                                                                                                                                                                                                     | M1<br>A1 cao                                             |
| (*)   | $\text{Alternatively, } v = \cos x, \text{ followed by } \sqrt{2}v = \sec \theta$                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | (9)                                                      |