Paper Reference(s)

9801/01

Edexcel

Advanced Extension Award

Friday 1 July 2005 – Morning
Time: 3 hours

Materials required for examination	Items included with question papers
Mathematical Formulae (Lilac or Green)	Nil
Graph paper (ASG2)	
Answer Book (AB16)	

Candidates may NOT use a calculator in answering this paper.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the paper title (Mathematics), the paper reference (9801), your surname, initials and signature.

Answers should be given in as simple a form as possible. e.g. $\frac{2\pi}{3}$, $\sqrt{6}$, $3\sqrt{2}$.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper.
The total mark for this paper is 100, of which 7 marks are for style, clarity and presentation.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N21152A

1. A point P lies on the curve with equation

$$x^2 + y^2 - 6x + 8y = 24.$$

Find the greatest and least possible values of the length OP, where O is the origin.

(6)

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2. Solve, for $0 < \theta < 2\pi$,

$$\sin 2\theta + \cos 2\theta + 1 = \sqrt{6} \cos \theta,$$

giving your answers in terms of π.

(8)

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3. Given that

$$\frac{\mathrm{d}}{\mathrm{d}x}(u\sqrt{x}) = \frac{\mathrm{d}u}{\mathrm{d}x} \times \frac{\mathrm{d}(\sqrt{x})}{\mathrm{d}x}, \quad 0 < x < \tfrac{1}{2},$$

where u is a function of x, and that $u = 4$ when $x = \tfrac{3}{8}$, find u in terms of x.

(9)

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4. A rectangle $ABCD$ is drawn so that A and B lie on the x-axis, and C and D lie on the curve with equation $y = \cos x$, $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$. The point A has coordinates $(p, 0)$, where $0 < p < \dfrac{\pi}{2}$.

(*a*) Find an expression, in terms of p, for the area of this rectangle.

(2)

The maximum area of $ABCD$ is S and occurs when $p = \alpha$. Show that

(*b*) $\dfrac{\pi}{4} < \alpha < 1$,

(6)

(*c*) $S = \dfrac{2\alpha^2}{\sqrt{(1+\alpha^2)}}$,

(2)

(*d*) $\dfrac{\pi^2}{2\sqrt{(16+\pi^2)}} < S < \sqrt{2}$.

(3)

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5. The point A has position vector $7\mathbf{i} + 2\mathbf{j} - 7\mathbf{k}$ and the point B has position vector $12\mathbf{i} + 3\mathbf{j} - 15\mathbf{k}$.

(*a*) Find a vector for the line L_1 which passes through A and B.

(2)

The line L_2 has vector equation

$$\mathbf{r} = -4\mathbf{i} + 12\mathbf{k} + \mu(\mathbf{i} - 3\mathbf{k}).$$

(*b*) Show that L_2 passes through the origin O.

(1)

(*c*) Show that L_1 and L_2 intersect at a point C and find the position vector of C.

(3)

(*d*) Find the cosine of $\angle OCA$.

(3)

(*e*) Hence, or otherwise, find the shortest distance from O to L_1.

(3)

(*f*) Show that $|\overrightarrow{CO}| = |\overrightarrow{AB}|$.

(2)

(*g*) Find a vector equation for the line which bisects $\angle OCA$.

(5)

6.

Figure 1

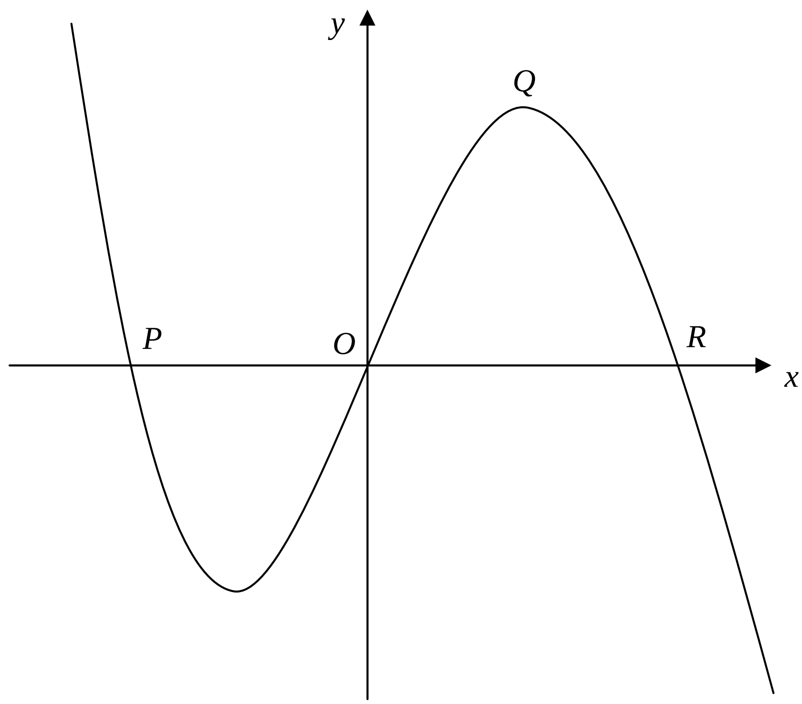

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = x(12 - x^2).$$

The curve cuts the x-axis at the points P, O and R, and Q is the maximum point.

(*a*) Find the coordinates of the points P, Q and R.

(4)

(*b*) Sketch, on separate diagrams, the graphs of

(i) $y = \mathrm{f}(2x)$,

(ii) $y = \mathrm{f}(|x| + 1)$,

indicating on each sketch the coordinates of any maximum points and the intersections with the x-axis.

(6)

Figure 2

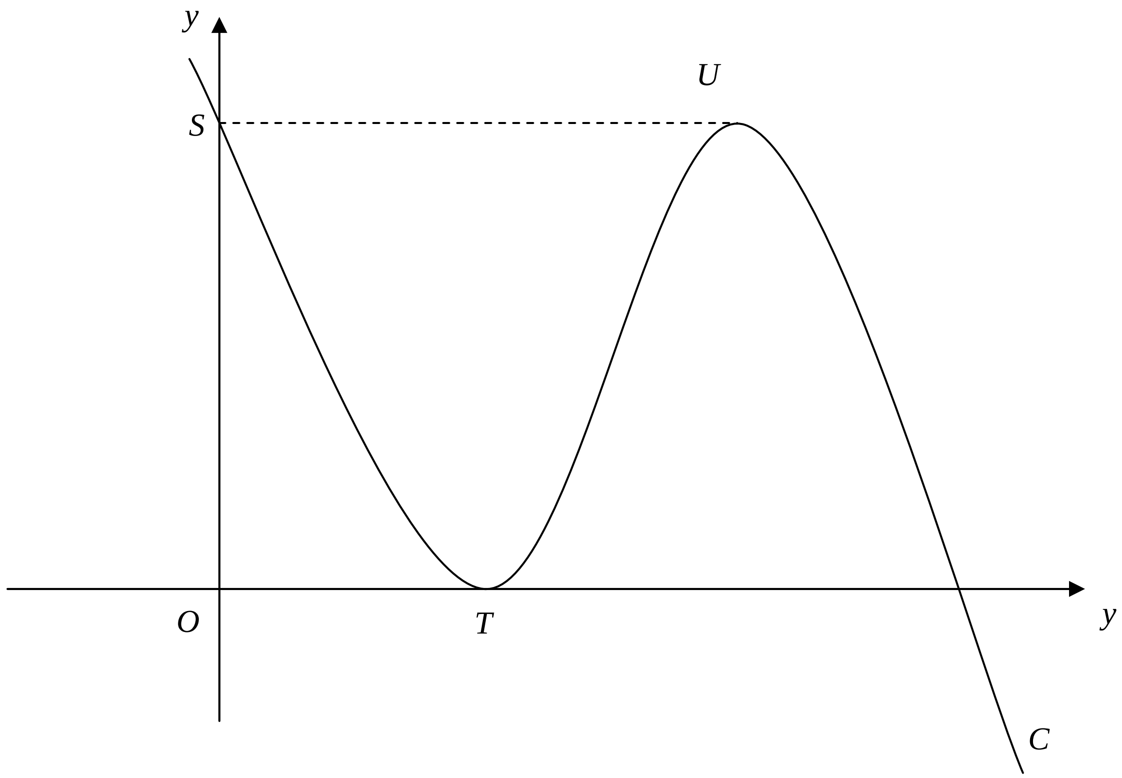

Figure 2 shows a sketch of part of the curve C, with equation $y = \mathrm{f}(x - v) + w$, where v and w are constants. The x-axis is a tangent to C at the minimum point T, and C intersects the y-axis at S. The line joining S to the maximum point U is parallel to the x-axis.

(c) Find the value of v and the value of w and hence find the roots of the equation

$$\mathrm{f}(x - v) + w = 0.$$

(9)

7. (*a*) Use the substitution $x = \sec\theta$ to show that

$$\int \sqrt{(x^2 - 1)}\ \mathrm{d}x$$

can be written as

$$\int \sec\theta \tan^2\theta\ \mathrm{d}\theta.$$

(3)

(*b*) Use integration by parts to show that

$$\int \sec\theta \tan^2\theta\ \mathrm{d}\theta = \frac{1}{2}\left[\sec\theta\tan\theta - \ln\left|\sec\theta + \tan\theta\right|\right] + \text{constant.}$$

(7)

(*c*) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \sin x \sqrt{(\cos 2x)}\ \mathrm{d}x$.

(9)

MARKS FOR STYLE, CLARITY AND PRESENTATION: 7 MARKS

TOTAL FOR PAPER: 75 MARKS

END