Kinematics

1) Given $v = u + at$ and $s = \frac{(u+v)t}{2}$. Derive $s = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2as$

Dynamics

2) A particle of mass 5kg is projected up a rough slope by a force D. The slope makes an angle of $40°$ with the horizontal. The coefficient of friction between the particle and the slope is 0.5, and that the particle accelerates at a rate of $3ms^{-2}$.

3 of the following 5 equations model the above situation correctly. Identify all three correct models. You may select no more than 3.

$$D - 5g\cos 50 \ -\frac{1}{2}R = 15$$

$$D - 5g\sin 50 - \frac{1}{2}R = 15$$

$$2D - 10g\sin 40 - 30 = R$$

$$D - 15 = 5g\cos 50 + \frac{1}{2}(5g\cos 40)$$

$$D - 15 = 5g\sin 40 + \frac{1}{2}(5g\cos 50)$$

Points of inflection

3) Dave claims that as $f''(a) = 0$ there is a point of inflection at $x = a$.

Give an example where this is not true. You must justify your answer.

Dave now claims that all points of inflection are stationary points.

Give an example where this is not true. You must justify your answer.

Logs

4) Starting from

$$\log A + \log B \equiv \log C$$

Prove using laws of indices that $C = AB$

Hence explain

$$B\log A \equiv \log A^B$$

Probability

5) Why does $P(A \cap B) = P(A)P(B)$ prove that two events are independent

Elementary trig

6) Using a right angled triangle ABC where $B\hat{A}C = \theta°, B\hat{C}A = 90$. Show that

$$\sin^2\theta + \cos^2\theta = 1$$

Indefintie integration

7) James has correctly calculated that

$$\int_0^5 f(x) = -12$$

James therefore concludes that area cannot be negative and that the area between $f(x)$ the x axis bounded by $x = 0$ and $x = 5$ must be 12. Explain why James might be wrong.

Reciprocal trig and pythaogrean idenitities

8) Starting from $\cos^2 x + \sin^2 x = 1$ Derive the Pythagorean identity for $\sec^2 x$

Differentiation

9) Given that $\frac{d}{dx}(e^x) = e^x$

Prove that $\frac{d}{dx}(\ln x) = \frac{1}{x}$

Givent that $\frac{d}{dx}(\ln x) = \frac{1}{x}$

Prove that $\frac{d}{dx}(a^x) = a^x \ln a$

10) Given that $\frac{d}{dx}(\sin x) = \cos x$ and $\frac{d}{dx}(\cos x) = -\sin x$

Use the quotient rule to show $\frac{d}{dx}(\tan x) = \sec^2 x$

Use the chain rule to show $\frac{d}{dx}(\sec x) = \sec x \tan x$

11) Show that $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$

Functions

12) When asked to solve $f(a) = f^{-1}(a)$ Gemma instead solved $f(a) = a$. Explain why this a correct method.

Radians

13) Given a sector of a circle of radius r and angle $\theta°$. Derive the formula for sector area and arc length in radians

Integration

14) Show that

$$\int \tan x \, dx = \ln \sec x + C$$

You must show all of your working

15) Look at the following proof

$$\int \sec x \, dx$$

$$= \int \sec x \frac{\sec x + \tan x}{\sec x + \tan x} \, dx$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$$

$$= \ln \sec x + \tan x + C$$

Explains why lines 1 and 2 are equivalent

Explain how line 4 is derived from line 3

Trig

16) Starting from the formulas for $\cos(A + B)$ and $\cos(A - B)$ show that

$$\sin 5x \cos 2x \equiv \frac{1}{2}(\sin 7x + \sin 3x)$$

Starting from $\cos 2x = \cos^2 x - \sin^2 x$ show that

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

Newton Raphson

17) Given $f(x) = \sin(x) - x, 0 \leq x < \frac{3\pi}{2}$. What value of x_1 will mean newton Raphson will not work.

R alpha

18) Given $(x) = 3 \sin x + 4 \cos x$. 2 of the following 6 statements are correct. Identify the correct statements

$$f(x) = 5 \sin(x + 0.927)$$

$$f(x) = 5 \sin(x + 0.644)$$

$$f(x) = 5\sin(x - 0.644)$$

$$f(x) = 5 \cos(x - 0.44)$$

$$f(x) = 5 \cos(x + 0.644)$$

$$f(x) = 5 \cos(x - 0.927)$$

19) Prove the formula for the sum of a geometric series

What condition is necessary when using the infinite sum of a geometric series.

Prove the formula for the infinite sum of a geometric sere

Prove the formula for the sum of an arithmetic series