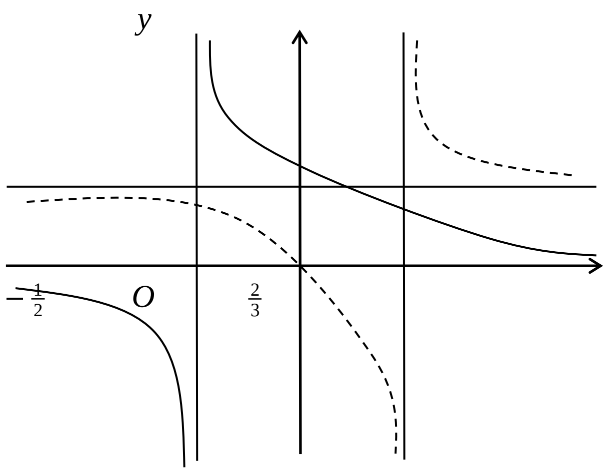
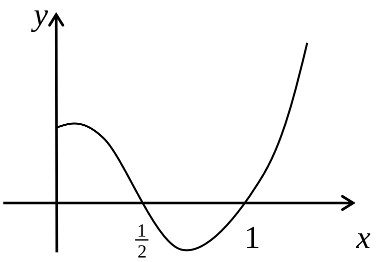
TT2 2017 mark scheme FP2 PRACTICE PAPER 3

1. (*a*)	$\frac{1}{2}a^2\int 1+\cos^2\theta+2\cos\theta \;\; d\theta$	M1 A1correct with limits
	$= \frac{1}{2}a^2\int 1+\frac{\cos 2\theta+1}{2}+2\cos\theta \; d\theta$	M1 A1
	$= 2\times\frac{1}{2}a^2\left[\theta+\frac{\sin 2\theta}{4}+\frac{\theta}{2}+2\sin\theta\right]_0^{\pi}$	A1
	$= a^2\left[\frac{3\pi}{2}\right] = \frac{3\pi a^2}{2}$	A1 **(6)**
(*b*)	$x = a\cos\theta + a\cos^2\theta$ $\quad$ $r\cos\theta$	M1
	$\frac{dx}{d\theta} = -a\sin\theta - 2a\cos\theta\sin\theta$	A1
	$\frac{dx}{d\theta} = 0 \Rightarrow \cos\theta = -\frac{1}{2}$ $\quad$ finding θ	M1
	$\theta = \frac{2\pi}{3}$ or $\theta = \frac{4\pi}{3}$	
	$r = \frac{a}{2}$ or $r = \frac{a}{2}$ $\quad$ finding r	M1
	A: $r = \frac{a}{2}$, $\theta = \frac{2\pi}{3}$	
	B: $r = \frac{a}{2}$, $\theta = \frac{-2\pi}{3}$ $\quad$ both A and B	A1 **(5)**
(*c*)	$x = -\frac{1}{4}a \quad \therefore\; WX = 2a + \frac{1}{4}a = 2\frac{1}{4}a$	M1 A1 **(2)**
(*d*)	$WXYZ = \frac{27\sqrt{3}a^2}{8}$	B1 ft **(1)**
(*e*)	Area $= \frac{27\sqrt{3}}{8}\times 100 - \frac{3\pi\times 100}{2} = 113.3 \text{ cm}^2$	M1 A1 **(2)**
		(16 marks)

2. (*a*)	$\frac{r^2-(r-1)^2}{r^2(r-1)^2} = \frac{2r-1}{r^2(r-1)^2}$	M1 A1 (2)
(*b*)	$\sum_{r=2}^{n}\frac{2r-1}{r^2(r-1)^2} = \sum_{r=2}^{n}\frac{1}{(r-1)^2}-\frac{1}{r^2}$	M1
	$= \frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{2^2}-\frac{1}{3^2}+\ldots+\frac{1}{(n-1)^2}-\frac{1}{n^2}$	M1
	$= 1-\frac{1}{n^2}$ $\quad$ (*)	A1 cso (3)

3.	Identifying as critical values $-\frac{1}{2}$, $\frac{2}{3}$	B1, B1
	Establishing there are no further critical values	
	Obtaining $2x^2 - 2x + 2$ or equivalent	M1
	$\Delta = 4 - 16 < 0$	A1
	Using exactly two critical values to obtain inequalities	M1
	$-\frac{1}{2} < x < \frac{2}{3}$	A1
		(6 marks)
Graphical alt.	Identifying $x = -\frac{1}{2}$ and $x = \frac{2}{3}$ as vertical asymptotes	B1, B1
	Two rectangular hyperbolae oriented correctly with respect to asymptotes in the correct half-planes.	M1
	Two correctly drawn curves with no intersections	A1
	As above	M1, A1
		

4. (*a*)	$\frac{dt}{dx} = 2x$ or equivalent	M1
	$I = \frac{1}{2}\int te^{-t}\,dt$ complete substitution	M1
	$= -te^{-t}\,dt + \frac{1}{2}\int e^{-t}\,dt$	M1 A1
	$= -\frac{1}{2}te^{-t} - \frac{1}{2}e^{-t}\ (+c)$	A1
	$= -\frac{1}{2}x^2e^{-x^2} - \frac{1}{2}e^{-x^2}\ (+c)$	A1 **(6)**
(*b*)	I.F. $= e^{\int \frac{3}{x}dx} = x^3$ (or multiplying equation by x^2)	B1
	$\frac{d}{dx}(x^3y) = x^3e^{-x^2}$ or $x^3y = \int x^3e^{-x^2}\,dx$	M1
	$x^3y = -\frac{1}{2}x^2e^{-x^2} - \frac{1}{2}e^{-x^2} + \underline{C}$	A1ft <u>A1</u> (4)
		(10 marks)
Alts (*a*)	(i) mark $t = -x^2$ similarly	M1
	(ii) $\int x^2.(xe^{-x^2})\,dx$ with evidence of attempt at integration by parts	M1
	$= x^2(-\frac{1}{2}e^{-x^2}) + \frac{1}{2}\int 2x.e^{-x^2}\,dx$	M1 A1 + A1
	$= -\frac{1}{2}x^2e^{-x^2} - \frac{1}{2}e^{-x^2}\ (+c)$	M1 A1 (6)
	(iii) $u = e^{-x^2}$, $\frac{du}{dx} = -2xe^{-x^2}$	M1
	$x^2 = \ln u$ hence $I = \int \frac{1}{2}\ln u\,du$	M1
	$= \frac{1}{2}u\ln u - \frac{1}{2}\int u.\frac{1}{u}\,du$	M1 A1
	$= \frac{1}{2}u\ln u - \frac{1}{2}u\ (+c)$	A1
	$= -\frac{1}{2}x^2e^{-x^2} - \frac{1}{2}e^{-x^2}\ (+c)$	A1 (6)
	(The result $\int \ln u\,du = u\ln u - u$ may be quoted, gaining M1 A1 A1 but must be completely correct.)	

5. (*a*)	$y' = 2kt.\mathrm{e}^{3t} + 3kt^2\,\mathrm{e}^{3t}$ use of product rule	M1
	$y'' = 2k\mathrm{e}^{3t} + 12kt\,\mathrm{e}^{3t} + 9t^2\,\mathrm{e}^{3t}$ product rule, twice	M1
	substituting $2k + 12kt + 9kt^2 - 12kt - 18kt^2 + 9kt^2 = 4$	M1
	$k = 2$	A1 (4)
(*b*)	Aux. eqn. (if used) $(m-3)^2 = 0$ $m = 3$, repeated	
	$y_{\text{C.F.}} = (A + Bt)\,\mathrm{e}^{3t}$ M1 required form (allow just written down)	M1 A1
	G.S. $y = (A + Bt)\,\mathrm{e}^{3t} + 2t^2\,\mathrm{e}^{3t}$ (ft on $2t^2\,\mathrm{e}^{3t}$)	A1 ft (3)
(*c*)	$t = 0, y = 3 \Rightarrow A = 3$	B1
	$y' = B\mathrm{e}^{3t} + 3(A + Bt)\,\mathrm{e}^{3t} + 4t\mathrm{e}^{3t} + 6t^2\mathrm{e}^{3t}$	M1
	$y' = 0, t = 0 \Rightarrow 1 = B + 3A \Rightarrow B = -8$	M1
	$y = (3 - 8t + 2t^2)\mathrm{e}^{3t}$	A1 (4)
(*d*)	 $\cup$ shape crossing +ve x-axis	B1
	$\frac{1}{2}$, 1	B1
	$y' = (-3 + 4t)\mathrm{e}^{3t} + 3(1 - 3t + 2t^2)\mathrm{e}^{3t} = 0$	
	$6t^2 - 5t = 0$	M1
	$t = \dfrac{5}{6}$	A1
	$y = -\dfrac{1}{9}\mathrm{e}^{2.5}$ (≈ -1.35) awrt -1.35	A1 (5)
		(16 marks)

6. (a) $(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$	M1
$(\cos\theta + i\sin\theta)^5 = \cos^5\theta + 5\cos^4\theta(i\sin\theta) + 10\cos^3\theta(i\sin\theta)^2$	
$+ 10\cos^2\theta(i\sin\theta)^3 + 5\cos\theta(i\sin\theta)^4 + (i\sin\theta)^5$	M1 A1
$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$	M1
$= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-2\cos^2\theta+\cos^4\theta)$	M1
$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ (*)	A1 cso
	(6)
(b) $\cos 5\theta = -1$ (or 1, or 0)	M1
$5\theta = (2n \pm 1)180° \Rightarrow \theta = (2n \pm 1)36°$	A1
$x = \cos\theta = -1, -0.309, 0.809$	M1 A1
	(4)
	[10]

Total 63 marks