# Year 2 Double Maths (Further Pure 2) Summer Assignment Part 1

Due first lesson in September 2017

1 Show that $\sum_{r=1}^{2n}(2r-1)^2 = \frac{2}{3}n(16n^2-1)$

2 Prove that $\sum_{r=1}^{n} r(r+1) = \frac{n}{3}(n+1)(n+2)$

3. The points $P(4at^2, 4at)$ and $Q(16at^2, 8at)$ lie on the parabola $C$ with equation $y^2 = 4ax$, where $a$ is a positive constant.

(a) Show that an equation of the tangent to $C$ at $P$ is $2ty = x + 4at^2$.

(b) Hence, write down the equation of the tangent to $C$ at $Q$.

The tangent to $C$ at $P$ meets the tangent to $C$ at $Q$ at the point $R$.

(c) Find, in terms of $a$ and $t$, the coordinates of $R$.

4. $3x\frac{\mathrm{d}y}{\mathrm{d}x} + y = x$

5. $x\frac{\mathrm{d}y}{\mathrm{d}x} + 4y = \frac{e^x}{x^2}$

## Second Order Differential Equations

6 Find the general solution of the following second order differential equations, leaving your answers in the appropriate form

a) $2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 2y = 0$

b) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0$

c) $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y = 0$

d) $4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0$

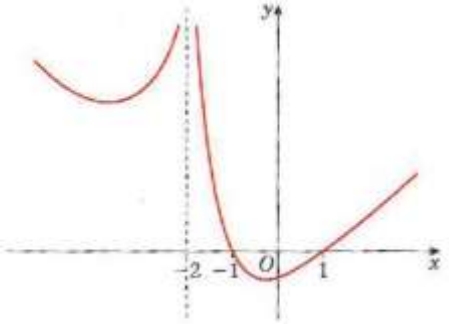
e) $5\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = 0$

f) $5\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 5y = 0$

7. Given that $z_1 = 1 + i$, $z_2 = 2 + i$ and $z_3 = 3 + i$, express the following in the form a + bi

(a) $\frac{z_1 z_2}{z_3}$ (b) $\frac{(z_2)^2}{z_1}$ (c) $\frac{2z_1 + 5z_3}{z_2}$

8




The figure above shows a sketch of the curve with equation $y = \frac{x^2 - 1}{|x+2|}, x \neq -2$

The curve crosses the $x$-axis at $x = -1$ and the line $x = -2$ is an asymptote of the curve.

(a) Use algebra to solve the equation $\frac{x^2 - 1}{|x+2|} = 3(1-x)$

(b) Hence, or otherwise, find the set of values of $x$ for which $\frac{x^2 - 1}{|x+2|} < 3(1-x)$

9 (a) Express $\frac{1}{r(r+2)}$ in partial fractions

(b) Hence prove, by the method of differences, that $\sum_{r=1}^{n} \frac{4}{r(r+2)} = \frac{n(3n+5)}{(n+1)(n+2)}$

(c) Find the value of $\sum_{r=50}^{100} \frac{4}{r(r+2)}$ , to 4 decimal places

10 $f(x) = \frac{2}{(x+1)(x+2)(x+3)}$

(a) Express f ($x$) in partial fractions

(b) Hence find $\sum_{r=1}^{n} f(r)$

11 (a) Show that $\frac{r^3 - r + 1}{r(r+1)} \equiv r - 1 + \frac{1}{r} - \frac{1}{r+1}$ , for $r \neq 0, -1$

(b) Find $\sum_{r=1}^{n} \frac{r^3 - r + 1}{r(r+1)}$ , expressing your answer as a single fraction in its simplest form

(1) Proof

(2) $\dfrac{1}{x+1}-\dfrac{1}{x+2}$

(3) $4ty = x + 16at^2$ (10c) $R(8at^2, 6at)$

(4) $y=\frac{1}{4}x+cx^{-\frac{1}{3}}$

(5) $y=\dfrac{1}{x^3}e^x-\dfrac{1}{x^4}e^x+\dfrac{c}{x^4}$

6

a) $y = Ae^{2x} + Be^{-\frac{1}{2}x}$

b) $y = e^{2x}(A\cos x + B\sin x)$

c) $y = e^{-4x}(Ax + B)$

d) $y = e^{\frac{1}{2}x}(Ax + B)$

e) $y = e^{-\frac{2}{5}x}\left(A\cos\frac{x}{5} + B\sin\frac{x}{5}\right)$

f) $y = e^{-\frac{3}{5}x}\left(A\cos\frac{4x}{5} + B\sin\frac{4x}{5}\right)$

(7) $\dfrac{3}{5}+\dfrac{4}{5}i$ (7b) $\dfrac{7}{2}+\dfrac{1}{2}i$ (7c) $\dfrac{41}{5}-\dfrac{3}{5}i$

8a) $-\frac{5}{2}, -\frac{7}{4}$ and 1 (8b) $x<-\frac{5}{2}, -\frac{7}{4}<x<1$

(9a) $\dfrac{1}{r(r+2)}=\dfrac{1}{2r}-\dfrac{1}{2(r+2)}$ (9c) 0.0398

(10a) $\mathrm{f}(x)=\dfrac{1}{x+1}-\dfrac{2}{x+2}+\dfrac{1}{x+3}$

(10b) $\dfrac{1}{6}-\dfrac{1}{n+2}+\dfrac{1}{n+3}$

(11b) $\dfrac{n(n^2+1)}{2(n+1)}$

# Year 2 Double Maths (Further Pure 2) Holiday Assignment Part 2

Due during first lesson September 2016

1. a) Find the particular solution of the differential equation

$$x^2 \frac{dy}{dx} = \tan y \text{ where } x = 1 \text{ when } y = \frac{\pi}{2}.$$

b) Find the value of $y$ when $x = 0.5$, giving your answer to 3 significant figures.

2 Find the sum of the series $\ln\frac{1}{2} + \ln\frac{2}{3} + \ln\frac{3}{4} + \ldots + \ln\frac{n}{n+1}$

3 Given that for all real values of $r$, $(2r+1)^3 - (2r-1)^3 = Ar^2 + B$ , where $A$ and $B$ are constants,

(a) Find the value of $A$ and the value of $B$

(b) Hence show that $\sum_{r=1}^{n} r^2 = \frac{1}{6} n(n+1)(2n+1)$

(c) Calculate $\sum_{r=1}^{40} (3r-1)^2$

$x$.

4 (a) On the same axes, sketch the graphs of $y = |x-5|$ and $y = |3x-2|$

(b) Find the set of values of $x$ for which $|x-5| < |x-3| > 2|x+1|$

5 (a) By expressing $\frac{2}{4r^2-1}$ in partial fractions, or otherwise, prove that $\sum_{r=1}^{n} \frac{2}{4r^2-1} = 1 - \frac{1}{2n+1}$

(b) Hence find the exact value of $\sum_{r=11}^{20} \frac{2}{4r^2-1}$

6 (a) Express $\frac{2}{(r+1)(r+3)}$ in partial fractions

(b) Hence prove that $\sum_{r=1}^{n} \frac{2}{(r+1)(r+3)} = \frac{n(5n+13)}{6(n+2)(n+3)}$

7 (a) Express $\frac{2r+3}{r(r+1)}$ in partial fractions

(b) Hence find $\sum_{r=1}^{n} \frac{2r+3}{r(r+1)} \bullet \frac{1}{3^r}$

8 (a) Express as a simplified single fraction $\frac{1}{(r-1)^2} - \frac{1}{r^2}$

(b) Hence prove, by the method of differences, that $\sum_{r=2}^{n} \frac{2r-1}{r^2(r-1)^2} = 1 - \frac{1}{n^2}$

9. At time *t* minutes an ink stain has area $A$ cm$^2$. When $t = 1, A = 1$ and the rate of increase of *A* is given by $t^2 \frac{\mathrm{d}A}{\mathrm{d}t} = A\sqrt{A}$.

a) Find an expression for *A* in terms of *t*.
b) Show that *A* never exceeds 4.

10 (a) Using the same axes, sketch the curve with equation $y = |x^2 - 6x + 8|$ and the line with equation $2y = 3x - 9$ . State the coordinates of the points where the curve and the line meet the *x*-axis.

(b) Use algebra to find the coordinates of the points where the curve and the line intersect and, hence, solve the inequality $2|x^2 - 6x + 8| > 3x - 9$

11 Find the set of real values of *x* for which

(a) $\frac{3x+1}{x-3} < 1$

(b) $\left|\frac{3x+1}{x-3}\right| < 1$

12 Use algebra to find the set of real values of *x* for which $|x-3| > 2|x+1|$

13 (a) Show that $\frac{r+1}{r+2} - \frac{r}{r+1} \equiv \frac{1}{(r+1)(r+2)}, r \in \mathbb{Z}^+$

(b) Hence, or otherwise, find $\sum_{r=1}^{n} \frac{1}{(r+1)(r+2)}$ , giving your answer as a single fraction in terms of *n*

14 $\mathrm{f}(r) = \frac{1}{r(r+1)}, r \in \mathbb{Z}^+$

(a) show that $f(r) - f(r+1) = \frac{k}{r(r+1)(r+2)}$ , stating the value of $k$

(b) Hence show, by the method of differences, that $\sum_{r=1}^{2n} \frac{1}{r(r+1)(r+2)} = \frac{n(2n+3)}{4(n+1)(2n+1)}$

15 (a) Express $\frac{1}{(x+1)(x+2)}$ in partial fractions

(b) Hence, or otherwise, show that $\sum_{r=1}^{n} \frac{2}{(r+1)(r+2} = \frac{n}{n+2}$

16 Solve, for $x$, the inequality $|5x+a| \leq |2x|$ , where $a > 0$

17 (a) Sketch, on the same axes, the graph of $y = |(x-2)(x-4)|$ , and the line with equation $y = 6-2x$

(b) Find the exact values of $x$ for which $|(x-2)(x-4)| = 6-2x$

(c) Hence solve the inequality $|(x-2)(x-4)| < 6-2x$

18 Use the given boundary conditions to the findthe particular solutions of these second order differential equations

a) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 10y = 0$, given that $y = 4$ and $\frac{dy}{dx} = 11$ when $x = 0$

b) $\frac{4d^2y}{dx^2} - 12\frac{dy}{dx} + 9y = 0$, given that $y = 4$ and $\frac{dy}{dx} = 3$ when $x = 0$

c) $9\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 4y = 0$, given that $y = 5$ and $\frac{dy}{dx} = -\frac{4}{3}$ when $x = 0$

d) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 0$, given that $y = 2$ and $\frac{dy}{dx} = -7$ when $x = 0$

e) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 8y = 0$, given that $y = 2$ when $x = 0$, and $y = e^{\frac{\pi}{2}}$ when $x = \frac{\pi}{4}$

19. Solve the differential equation, giving $y$ in terms of $x$, where $x^3\frac{\mathrm{d}y}{\mathrm{d}x} - x^2y = 1$ and $y = 1$ at $x = 1$.

20. a) Find the general solution of the differential equation $\cos x\frac{\mathrm{d}y}{\mathrm{d}x} + y = 1, -\frac{\pi}{2} < x < \frac{\pi}{2}$.

b) Find the particular solution which satisfies the condition that $y = 2$ at $x = 0$.

**Challenge**

Find the set of values of $x$ for which $-1 < \dfrac{2-x}{2+x} \le 1$

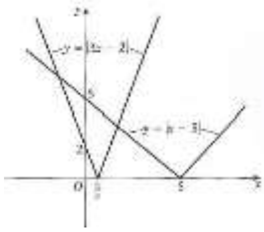
**Answers**

(1a) $\ln \sin y = 1 - \dfrac{1}{x}$ (1b) 0.377

(2) − ln($n$ + 1)

(3c) 194 380

(4a)



(4b) $x < -\frac{2}{3}, x > \frac{7}{4}$

(5b) $\dfrac{20}{861}$ (16a) $A$ = 24, $B$ = 2

(6a) $\dfrac{2}{(r+1)(r+3)} = \dfrac{1}{r+1} - \dfrac{1}{r+3}$

(7a) $\dfrac{2r+3}{r(r+1)} = \dfrac{3}{r} - \dfrac{1}{r+1}$ (7b) $1 - \dfrac{1}{3^n(n+1)}$

(8a) $\dfrac{2r-1}{r^2(r-1)^2}$

(9a) $A = \dfrac{4t^2}{(1+t)^2}$

(9b) since for $t > 0, t < (1+t)$ so $A < 4$.

(10a) Curve = (2, 0) and (4, 0), Line = (3, 0)

(10b) $A = \left(\frac{7}{2}, \frac{3}{4}\right)$ , $B$ = (5, 3)

(10c) $x < 3\frac{1}{2}, x > 5$

(11a) $-2 < x < 3$ (13b) $-2 < x < \frac{1}{2}$

(12) $-5 < x < \frac{1}{3}$

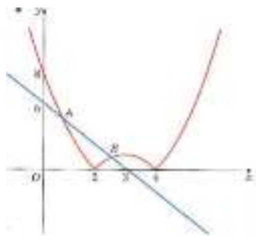
(13b) $\dfrac{n}{2(n+2)}$

(14a) $k$ = 2

(14a) $\dfrac{1}{(x+1)(x+2)} = \dfrac{1}{x+1} - \dfrac{1}{x+2}$

(16) $-\frac{1}{3}a \le x \le \frac{1}{7}a$

(17a)



(17b) $2 - \sqrt{2}$ and $4 - \sqrt{2}$ (17c) $2 - \sqrt{2} < x < 4 - \sqrt{2}$

(18

a) $y = e^{5x} + 3e^{2x}$

b) $y = e^{\frac{3x}{2}}(4 - 3x)$

c) $y = e^{-\frac{2x}{3}}(2x + 5)$

d) $y = e^{-2x}(2\cos 3x - \sin 3x)$

e) $y = e^{-3x}(2\cos 2x - \sin 2x)$

(19) $y = \dfrac{-1}{3x^2} + \dfrac{4x}{3}$

(20a) $y = 1 + \dfrac{c}{\sec x + \tan x}$

(20b) $y = 1 + \dfrac{1}{\sec x + \tan x}$ or $y = 1 + \dfrac{\cos x}{1 + \sin x}$

**Challenge Question**

$x \ge 0$