A2 Maths Continuing With Confidence Booklet

Name: ……………………………………………………………

Welcome back to BHASVIC Maths. We are an Outstanding Department and we aim for you to be outstanding too! This booklet has been designed to help you to feel confident in starting your second year, by revising everything you have done up to this point. Be sure to complete it all and bring it to your first lesson!

CONTENTS

Please read the below carefully before you start this booklet

Things to do before your first lesson back

1. You must complete **all** of this booklet and bring it to your **first lesson**. Your teacher will expect this to be **100% complete and correct** when you arrive. Write all your working in the booklet.

2. Each section begins with exercises involving basic skills, and the final exercise in each section will be exam questions to really test whether you know what you are doing.

3. Check all of your answers to the questions against those on the answer sheet (Page 3) and tick them off as you go.

4. Get help when you are stuck! Maths can be tough and getting stuck is normal. What makes a successful BHASVIC maths student is one who proactively seeks help to solve problems.

   How to get help: Watch the videos on any concept you need help with and join BHASVIC Maths Facebook to ask for advice. You could **attend the support sessions on August 31st, September 3rd, and September 4th** after enrolment.

5. Make sure you are confident with all of the concepts in this booklet. Your CWC test will be in class w/b 24th September on the topics in this booklet to assess your skills. If you have studied this booklet properly, there is no reason why you cannot do well in this test.

SECTION 1, 2, 3, 4, 5, 6, 7, 8, 9 & 10 ANSWERS

SECTION 1 – USING INDICES – DIFFERENTIATION & INTEGRATION

EX 1A: 1) $2x - \frac{2}{x^2}$ 2) $-2x^{-3} - x^{-2}$ 3) $-\frac{1}{x^2} + \frac{14}{x^3} - \frac{12}{x^4}$

EX 1B: 1) 4 2) $-\frac{3}{2}$ 3) $\frac{1}{54}$ 4) $2x^{\frac{2}{3}}$ 5) $2x^{-\frac{3}{2}} + 4x^{-2}$

EX 1C: 1) $\frac{2}{3}x^{\frac{3}{2}} + \frac{3}{2}x^{\frac{2}{3}} + c$ 2) $-\frac{2}{x^2} + \frac{1}{x} - \frac{x^3}{3} + c$ 3) $\frac{4}{7}x^{\frac{7}{2}} - \frac{5}{3}x^{\frac{3}{5}} + c$

EX 1D: 1) $10 - \frac{16\sqrt{2}}{3}$ 2) $130\frac{4}{5}$ 3) $-\frac{62}{81}$

EX 1E:

1)

$\frac{6x^2-1}{2\sqrt{x}} = 3x^{\frac{3}{2}} - \frac{1}{2}x^{-\frac{1}{2}}$ M1 A1

$\frac{d}{dx}(3x^{\frac{3}{2}} - \frac{1}{2}x^{-\frac{1}{2}}) = \frac{9}{2}x^{\frac{1}{2}} + \frac{1}{4}x^{-\frac{3}{2}}$ M1 A2 **(5)**

2)

(a) $\frac{dy}{dx} = 1 - 3x^{-2}$ M1 A1

grad $= 1 - 3(1)^{-2} = 1 - 3 = -2$ A1

(b) $x = 1 \therefore y = 4$

grad $= \frac{-1}{-2} = \frac{1}{2}$ M1 A1

$\therefore y - 4 = \frac{1}{2}(x - 1)$ M1

$y = \frac{1}{2}x + \frac{7}{2}$ A1

(c) $x + \frac{3}{x} = \frac{1}{2}x + \frac{7}{2}$

$2x^2 + 6 = x^2 + 7x$ M1

$x^2 - 7x + 6 = 0, \quad (x - 1)(x - 6) = 0$ M1

$x = 1$ (at P), 6 A1

$\therefore (6, 6\frac{1}{2})$ A1 **(11)**

3)

(a) $2x^2 + 6x + 7 = 2x + 13$

$x^2 + 2x - 3 = 0$ M1

$(x + 3)(x - 1) = 0$ M1

$x = -3, 1$ A1

$\therefore (-3, 7), (1, 15)$ A1

(b) area under curve $= \int_{-3}^{1} (2x^2 + 6x + 7)\ dx$

$= [\frac{2}{3}x^3 + 3x^2 + 7x]_{-3}^{1}$ M1 A2

$= (\frac{2}{3} + 3 + 7) - (-18 + 27 - 21) = 22\frac{2}{3}$ M1

area of trapezium $= \frac{1}{2} \times (7 + 15) \times 4 = 44$ B1

shaded area $= 44 - 22\frac{2}{3} = 21\frac{1}{3}$ M1 A1 **(11)**

SECTION 2 – LOGS & EXPONENTIALS

EX 2A: 1) $x = 15$ 2) $x = 9$ 3) $x = 625$ 4) $x = \frac{1}{21}$ 5) $\frac{10}{11}$ 6) $x = 0.297$

EX 2B: 1) $x = ln5, x = \ln\left(\frac{1}{3}\right)$ 2) $x = e^3, x = e^{-5}$ 3) $x = \frac{1}{2}ln2, x = 0$ 4) $x = e^6, x = e^{-2}$

EX 2C:

1)

(a) *(i)* $= 2\log_3 x = 2t$ M1 A1

(ii) $= \frac{\log_3 x}{\log_3 9} = \frac{\log_3 x}{2} = \frac{1}{2}t$ M1 A1

(b) $2t - \frac{1}{2}t = 4$

$t = \frac{8}{3}$ M1

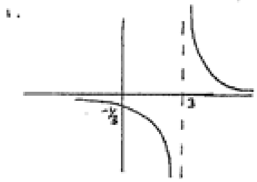
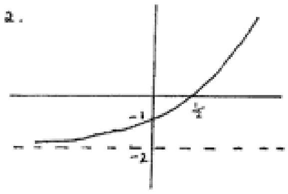
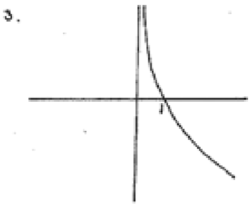
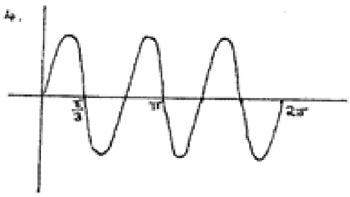
$\log_3 x = \frac{8}{3}, \quad x = 3^{\frac{8}{3}} = 18.7$ M1 A1 **(7)**

2)

$e^{2y} - x + 2 = 0 \quad \Rightarrow \quad e^{2y} = x - 2$		
$2y = \ln(x-2)$	M1	
sub. $\Rightarrow \quad \ln(x+3) - \ln(x-2) - 1 = 0$	A1	
$\ln \frac{x+3}{x-2} = 1$	M1	
$\frac{x+3}{x-2} = e$	A1	
$x + 3 = e(x-2)$	M1	
$3 + 2e = x(e-1)$	M1	
$x = \frac{2e+3}{e-1} = 4.91$ (2dp), $y = \frac{1}{2}\ln\left(\frac{2e+3}{e-1} - 2\right) = 0.53$ (2dp)	A2	**(8)**

SECTION 3 – GRAPH SKETCHING

EX 3A:

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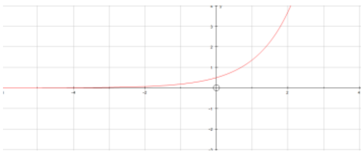
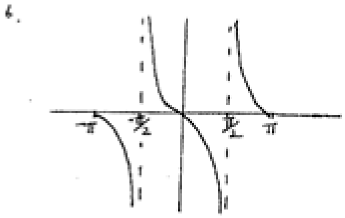
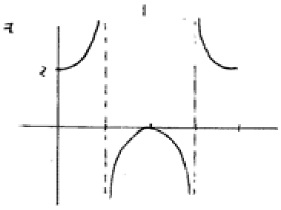
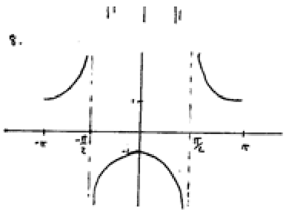

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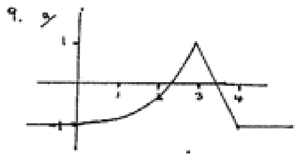
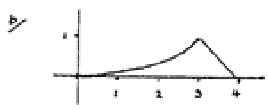
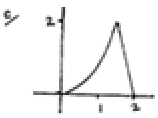
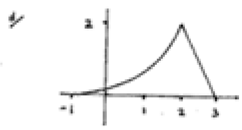
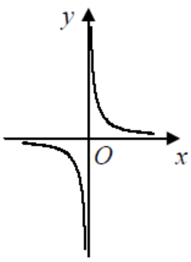





EX 3C:

1)

(a) stretch by factor of 3 in y-direction about x-axis — B2
or stretch by factor of 3 in x-direction about y-axis

(b)



asymptotes: $x = 0$ and $y = 0$ — B2, B1

(c) $\frac{3}{x} = c - 3x$

$3 = cx - 3x^2$ — M1

$3x^2 - cx + 3 = 0$

tangent $\therefore$ equal roots, $b^2 - 4ac = 0$

$(-c)^2 - (4 \times 3 \times 3) = 0$ — M1 A1

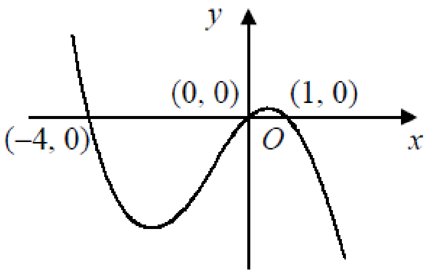
$c^2 = 36, \quad c = \pm 6$ — A1 **(9)**

2)

(a) $= x(4 - 3x - x^2)$ M1

$= x(1 - x)(4 + x)$ M1 A1

(b)



B3

(6)

3)

quadratic, coeff of $x^2 = 1$, minimum $(-2, 5)$

$\therefore\ y = (x + 2)^2 + 5$ M1 A1

$= x^2 + 4x + 9, \quad a = 4, b = 9$ M1 A1 **(4)**

SECTION 4 – TRIGONOMETRY

EX 4A: 1) $\frac{\pi}{4}, \frac{5\pi}{4}$ 2) $0.491, 2.062, 3.633, 5.204$ 3) $\frac{7\pi}{12}, \frac{23\pi}{12}$ 4) $\frac{\pi}{2}, \frac{3\pi}{2}$ 5) $\frac{\pi}{3}, \frac{2\pi}{3}$

6) $\frac{\pi}{4}, \frac{3\pi}{4}$ 7) $0.308, 2.834$ 8) $\frac{\pi}{2}$ 9) $\frac{\pi}{3}, \frac{5\pi}{3}$ 10) $\frac{\pi}{6}, \frac{7\pi}{6}$ 11) $-\pi, 0, \pi, -\frac{3\pi}{4}, \frac{\pi}{4}$ 12) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

EX 4B: 1) $1.28, 5.01$ 2) $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$ 3) $\frac{\pi}{2}, \frac{3\pi}{2}, 3.39, 6.03$

4) $\frac{\pi}{3}, \frac{5\pi}{3}, 1.32, 4.97$ 5) $0.553, 2.12, 3.69, 5.27$ 6) $0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, 2\pi$

7) $\frac{\pi}{3}, \frac{5\pi}{3}, 1.91, 4.37$ 8) $0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi$

EX 4C: TRIG PROOFS

EX 4D:

1)

$\tan^2\theta = \frac{1}{3}$ M1

$\tan\theta = \pm\frac{1}{\sqrt{3}}$ A1

$\theta = \frac{\pi}{6}, \frac{\pi}{6} - \pi$ or $\pi - \frac{\pi}{6}, -\frac{\pi}{6}$ B1 M1

$\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$ A2 **(6)**

2)

(a) *(i)* LHS $= \sin x \cos 30 + \cos x \sin 30 + \sin x \cos 30 - \cos x \sin 30$ M1 A1

$= 2 \sin x \cos 30 = \sqrt{3} \sin x \quad [a = \sqrt{3}\,]$ A1

(ii) let $x = 45$, $\sin 75 + \sin 15 = \sqrt{3} \sin 45 = \sqrt{3} \times \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{6}$ M2 A1

(b) $2(\text{cosec}^2 y - 1) + 5 \text{ cosec } y + \text{cosec}^2 y = 0$ M1

$3 \text{ cosec}^2 y + 5 \text{ cosec } y - 2 = 0, \quad (3 \text{ cosec } y - 1)(\text{cosec } y + 2) = 0$ M1

$\text{cosec } y = -2$ or $\frac{1}{3}$ (no solutions) A1

$\sin y = -\frac{1}{2}$

$y = 180 + 30, 360 - 30$ B1 M1

$y = 210, 330$ A1 **(12)**

3)

(a) $= \text{f}(\frac{1}{2}) = -\frac{5}{2}$ M1 A1

(b) $\text{gf}(x) = \frac{2}{(3x-4)+3} = \frac{2}{3x-1}$ M1 A1

$\therefore \frac{2}{3x-1} = 6$

$2 = 6(3x - 1)$ M1

$x = \frac{4}{9}$ A1 **(6)**

SECTION 5 – DIFFERENTIATION

EX 5A: 1) $4x(x^2-3)$ 2) $-10(3-x)^9$ 3) $-15(5x+2)^{-4}$ 4) $40(1-x)^{-5}$

5) $(15x^2-4x+1)e^{5x^3-2x^2+x-2}$ 6) $cosx\,e^{sinx}$ 7) $3\sec^2(3x+1)$

8) $-(8x+1)\sec(4x^2+x)\tan(4x^2+x)$ 9) $\frac{8}{3}cosec^2\left(\frac{2x}{3}\right)$ 10) $3\sin^2 x\ cosx$

11) $1+\frac{cosx}{2\sqrt{sinx}}$ 12) $3(sin\ 2x)^{\frac{1}{2}}\cos 2x$ 13) $-12cosec^3 4x cot4x$

14) $-10\sec^2(5x+1)\tan^{-\frac{1}{2}}(5x+1)$ 15) $\frac{15x^2+4x+1}{5x^3+2x^2+x-2}$ 16) $\frac{secx+tanx}{secx}$

17) $3^x ln3$ 18) $\sec^2 x\, 4^{tanx} ln4$

EX 5B: 1) $(3x+1)^3(15x-35)$ 2) $\frac{x^2}{\sqrt{4x-1}}(14x-3)$ 3) $x^2\cos x+2x\,sinx$

4) $\sec(x)\tan(x)\,cosec(3x)-3\sec(x)\,cosec(3x)\cot(3x)$ 5) $10xe^{5x^2}cot3x-3e^{5x^2}cosec^2 3x$

6) $15x^2\ln(2x+1)+\frac{6x^5}{2x+1}$ 7) $\frac{-x^3-9x^2}{(x-3)^5}$ 8) $\frac{2x+3}{(4x+3)^{\frac{3}{2}}}$ 9) $\frac{2x\cos 2x-2\sin 2x}{x^3}$ 10) $-cosec^2 x$

EX 5C:

1)

$\frac{dx}{dy}=2\sec y\times\sec y\tan y+\sec^2 y=\sec^2 y(2\tan y+1)=\frac{2\tan y+1}{\cos^2 y}$ M1 A1

$\frac{dy}{dx}=1\div\frac{dx}{dy}=\frac{\cos^2 y}{2\tan y+1}$ M1 A1 **(4)**

2)

(a) $=\frac{1}{3x-2}\times 3=\frac{3}{3x-2}$ M1 A1

(b) $=\frac{2\times(1-x)-(2x+1)\times(-1)}{(1-x)^2}=\frac{3}{(1-x)^2}$ M1 A2

(c) $=\frac{3}{2}x^{\frac{1}{2}}\times e^{2x}+x^{\frac{3}{2}}\times 2e^{2x}=\frac{1}{2}x^{\frac{1}{2}}e^{2x}(3+4x)$ M1 A2 **(8)**

3)

(a) $\frac{d}{dx}(\tan x)=\frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$

$=\frac{\cos x\times\cos x-\sin x\times(-\sin x)}{\cos^2 x}$ M1 A1

$=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}=\frac{1}{\cos^2 x}=\sec^2 x$ M1 A1

(b) $\frac{dy}{dx}=2\times\tan x+2x\times\sec^2 x=2\tan x+2x\sec^2 x$ M1 A1

$x=\frac{\pi}{4},\ y=\frac{\pi}{2},\ \text{grad}=2+\pi$ B1

$\therefore\ y-\frac{\pi}{2}=(2+\pi)(x-\frac{\pi}{4})$ M1

at P, $x=0$

$\therefore\ y=\frac{\pi}{2}-\frac{\pi}{4}(2+\pi)=-\frac{1}{4}\pi^2$ M1 A1 **(10)**

SECTION 6 – INTEGRATION

EX 6A: 1) $5e^x+4cosx+\frac{1}{2}x^4+c$ 2) $3secx-2\ln|x|+c$ 3) $\frac{1}{2}\ln|x|-2cotx+c$

4) $-2cosecx-tanx+c$ 5) $\frac{1}{42}(3x^2-5)^7+c$ 6) $\frac{1}{24}(4x^2-5)^9+c$

7) $-\frac{1}{4}(2x^2-7)^{-4}+c$ 8) $-\frac{3}{20}(1-5x^4)^{-2}+c$ 9) $\frac{1}{15}\sin^5(3x+2)+c$

10) $-\frac{1}{4}\cos^{10}2x + c$ 11) $-\frac{7}{45}\tan^5 9x + c$ 12) $\frac{1}{16}cosec^6 8x + c$
13) $-\frac{1}{2}(1 + tan2x)^{-2} + c$ 14) $\frac{1}{4}e^{8x^2-3} + c$ 15) $2\ln|x^2 + 3x| + c$ 16) $\frac{1}{2}\ln|1 + tan2x| + c$

EX 6B: 1) $-2cotx - x + 2cosecx + c$ 2) $2x - tanx + c$ 3) $-\frac{1}{2}cosec2x + c$
4) $\frac{1}{8}x - \frac{1}{32}sin4x + c$ 5) $-3x + \frac{3}{14}sin14x + c$ 6) $5x + \frac{5}{4}sin4x + c$

EX 6C: 1) Partial Fractions = $\frac{3}{x+1} - \frac{1}{x+4}$ Integral = $3\ln|x + 1| - \ln|x + 4| + c$

2) Partial Fractions = $-\frac{2}{x} + \frac{4}{x-3} + \frac{2}{(x-3)^2}$ Integral = $-2\ln|x| + 4\ln|x - 3| - 2(x - 3)^{-1} + c$
3) Partial Fractions = $\frac{1}{x+2} + \frac{1}{(x+2)^2} + \frac{1}{(x+3)^3}$ Integral = $\ln|x + 2| - (x + 2)^{-1} - \frac{1}{2}(x + 2)^{-2} + c$

4) Partial Fractions = $4 + \frac{2}{x-1} + \frac{3}{x+4}$ Integral = $4x + 2\ln|x - 1| + 3\ln|x + 4| + c$

EX 6D:
1)

(a) $15 - 17x \equiv A(1-3x)^2 + B(2+x)(1-3x) + C(2+x)$
$x = -2 \Rightarrow 49 = 49A \Rightarrow A = 1$ B1
$x = \frac{1}{3} \Rightarrow \frac{28}{3} = \frac{7}{3}C \Rightarrow C = 4$ B1
coeffs $x^2 \Rightarrow 0 = 9A - 3B \Rightarrow B = 3$ M1 A1

(b) $= \int_{-1}^{0} \left(\frac{1}{2+x} + \frac{3}{1-3x} + \frac{4}{(1-3x)^2}\right) dx$
$= [\ln|2+x| - \ln|1-3x| + \frac{4}{3}(1-3x)^{-1}]_{-1}^{0}$ M1 A3
$= (\ln 2 + 0 + \frac{4}{3}) - (0 - \ln 4 + \frac{1}{3})$ M1
$= 1 + \ln 8$ M1 A1 **(11)**

2)
$= \int (\text{cosec}^2 2x - 1)\ dx$ M1 A1
$= -\frac{1}{2}\cot 2x - x + c$ M1 A1 **(4)**

3)
(a) $2x^3 - 5x^2 + 6 \equiv (Ax + B)x(x-3) + C(x-3) + Dx$ M1
$x = 0 \Rightarrow 6 = -3C \Rightarrow C = -2$
$x = 3 \Rightarrow 15 = 3D \Rightarrow D = 5$ A1
coeffs $x^3 \Rightarrow A = 2$ B1
coeffs $x^2 \Rightarrow -5 = B - 3A \Rightarrow B = 1$ M1 A1

(b) $= \int_{1}^{2} (2x + 1 - \frac{2}{x} + \frac{5}{x-3})\ dx$
$= [x^2 + x - 2\ln|x| + 5\ln|x-3|]_{1}^{2}$ M1 A2
$= (4 + 2 - 2\ln 2 + 0) - (1 + 1 + 0 + 5\ln 2)$ M1
$= 4 - 7\ln 2$ A1 **(10)**

SECTION 7 – BINOMIAL EXPANSION

EX 7A: 1) $243 - 1620x + 4320x^2 - 5760x^3 + 3840x^4 - 1024x^5$ 2) $1679616 - 4478976x + 5225472x^2$ 3) $a^{10} - 5a^8b^2 + 10a^6b^4 - 10a^4b^6 + 5a^2b^8 - b^{10}$ 4) $\frac{1}{x^3} + 3 + 3x^3 + x^6$

EX 7B: 1) $\frac{1}{4} + \frac{1}{4}x + \frac{3}{16}x^2 + \frac{1}{8}x^3$ 2) $3 - \frac{1}{3}x - \frac{1}{54}x^2 - \frac{1}{486}x^3$ 3) $\frac{1}{3} + \frac{1}{9}x + \frac{1}{27}x^2 + \frac{1}{81}x^3$
4) $\frac{1}{9} + \frac{4}{27}x - \frac{1}{9}x^2 + \frac{14}{243}x^3$

EX 7C: 1)

(a) $= 1 + 8(3x) + \binom{8}{2}(3x)^2 + \binom{8}{3}(3x)^3 + \ldots$ M1 A1
$= 1 + 24x + 252x^2 + 1512x^3 + \ldots$ M1 A1

(b) $x = 0.001$ B1
$(1.003)^8 \approx 1 + 0.024 + 0.000\,252 + 0.000\,001\,512$ M1
$= 1.024\,253\,5$ (8sf) A1 **(7)**

2)

(a) $f(\frac{1}{10}) = \frac{3}{\sqrt{1-\frac{1}{10}}} = \frac{3}{\sqrt{\frac{9}{10}}} = \frac{3}{(\frac{3}{\sqrt{10}})} = \sqrt{10}$ M1 A1

(b) $= 3(1-x)^{-\frac{1}{2}} = 3[1 + (-\frac{1}{2})(-x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(-x)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3\times 2}(-x)^3 + \ldots]$ M1

$= 3 + \frac{3}{2}x + \frac{9}{8}x^2 + \frac{15}{16}x^3 + \ldots$ A2

(c) $\sqrt{10} = f(\frac{1}{10}) \approx 3 + \frac{3}{20} + \frac{9}{800} + \frac{15}{16000} = 3.1621875$ (8sf) B1

(d) $= \frac{\sqrt{10} - 3.1621875}{\sqrt{10}} \times 100\% = 0.003\%$ (1sf) M1 A1 **(8)**

SECTION 8 – FUNCTIONS

EX 8A: 1) $f(x) \geq 2$ 2) $f(x) \geq 9$ 3) $0 \leq f(x) \leq 2$ 4) $f(x) \geq 0$ 5) $f(x) \geq 1$ 6) $f(x) \in R$

EX 8B: 1) $3x^2 + 14$ 2) $9x^2 + 12x + 8$ 3) $9x + 8$ 4) $b = \pm 4$

EX 8C: 1) $g^{-1}(x) = \frac{1}{x}$, $domain\ x \in R, x \leq \frac{1}{3}$, $Range\ g^{-1}(x) \geq 3$

2) $g^{-1}(x) = \frac{x+1}{2}$, $domain\ x \in R, x \geq -1$, $Range\ g^{-1}(x) \geq 0$

3) $g^{-1}(x) = \frac{2x+3}{x}$, $domain\ x \in R, x > 0$, $Range\ g^{-1}(x) > 2$

4) $g^{-1}(x) = x^2 + 3$, $domain\ x \in R, x \geq 2$, $Range\ g^{-1}(x) \geq 7$

EX 8D:

1)

(a) $= f(\frac{1}{2}) = -\frac{5}{2}$ M1 A1

(b) $gf(x) = \frac{2}{(3x-4)+3} = \frac{2}{3x-1}$ M1 A1

$\therefore \frac{2}{3x-1} = 6$

$2 = 6(3x-1)$ M1

$x = \frac{4}{9}$ A1 **(6)**

2)

(a) $= 2[x^2 + 2x] + 2 = 2[(x+1)^2 - 1] + 2$ M1

$= 2(x+1)^2$ A1

(b) translation by 1 unit in negative x direction

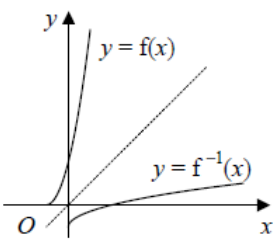
stretch by scale factor of 2 in y direction (either first) B3

(c) $y = 2(x+1)^2$, $\frac{y}{2} = (x+1)^2$ M1

$x + 1 = \pm\sqrt{\frac{y}{2}}$ M1

$x = -1 \pm \sqrt{\frac{y}{2}}$ (domain $\Rightarrow$ +), $\therefore\ f^{-1}(x) = -1 + \sqrt{\frac{x}{2}}$, $x \in \mathbb{R}$, $x \geq 0$ A2

(d)



B3

$y = f^{-1}(x)$ is reflection of $y = f(x)$ in line $y = x$ B1

(13)

SECTION 9 – STATISTICS - CODING

EX 9: 1) 22.9 2) 416 mm

SECTION 10 – MECHANICS – SLOPES

EX 10A: 1) $P = 7.07\ \ Q = 7.07$ 2) $P = 4.73\ \ Q = 4.20$ 3) $P = 9.24\ \ Q = 4.62$ 4) $P = 3.00\ \ Q = 0.657$

EX 10B: 1) a) $1.96 m/s^2$ b) $3.9m\ down\ plane$ 2) $7.1 m/s^2\ up\ the\ plane$

SECTION 1 – USING INDICES – DIFFERENTIATION & INTEGRATION

WRITE YOUR ANSWERS DIRECTLY INTO THIS BOOKLET, AND TICK THE BOXES WHEN YOU HAVE CHECKED THAT YOU ARE CORRECT

EXERCISE 1A – SIMPLIFYING INDICES TO DIFFERENTIATE

Find $f'(x)$ by manipulating indices i.e. **DO NOT USE** product rules, chain rules, or quotient rules.

Need help?

1) $f(x)=\dfrac{x^3+2}{x}$	2) $f(x)=x^{-2}(1+x)$	3) $f(x)=\dfrac{x^2-7x+4}{x^3}$

EXERCISE 1B – BASIC TANGENT PROBLEMS

Find the exact gradient of the tangent at the given point.

Need help?

1) $y=\dfrac{3x-1}{x}$ where $x=\dfrac{1}{2}$	2) $y=2\sqrt{x}\left(1-\sqrt{x}\right)$ where $x=4$	3) $y=\dfrac{\sqrt{x}-1}{\sqrt{x}}$ where $x=9$

EXERCISE 1C – MANIPULATING INDICES FOR BASIC INDEFINITE INTEGRALS

By manipulating indices, Integrate the following functions with respect to *x*

Need help?

1) $\sqrt{x}+\frac{1}{\sqrt[3]{x}}$	2) $\frac{4}{x^3}-\frac{1}{x^2}-x^2$	3) $2x^{\frac{5}{2}}-x^{-\frac{2}{5}}$

EXERCISE 1D – MANIPULATING INDICES FOR BASIC DEFINITE INTEGRALS

By manipulating indices, Integrate the following functions with respect to *x*

Need help?

1) $\int_0^2 \left(x^{\frac{1}{2}}-2\right)^2 \, \mathrm{d}x$	2) $\int_4^9 x^{\frac{1}{2}}\left(2x-3\right) \, \mathrm{d}x$	3) $\int_{-3}^{-1} \frac{x-1}{x^4} \, \mathrm{d}x$

EXERCISE 1E – EXAM PROBLEMS – BASIC DIFFERENTIATION & INTEGRATION

As stated at the front of the booklet, you will be given exam questions at the end of each section to really test whether you know what you are doing. Watch the videos again if you get stuck.

QUESTION 1

Differentiate with respect to x

$$\frac{6x^2 - 1}{2\sqrt{x}}.$$

(5)

QUESTION 2

A curve has the equation $y = x + \frac{3}{x}$, $x \neq 0$.

The point P on the curve has x-coordinate 1.

(a) Show that the gradient of the curve at P is -2. **(3)**

(b) Find an equation for the normal to the curve at P, giving your answer in the form $y = mx + c$. **(4)**

(c) Find the coordinates of the point where the normal to the curve at P intersects the curve again. **(4)**

QUESTION 3

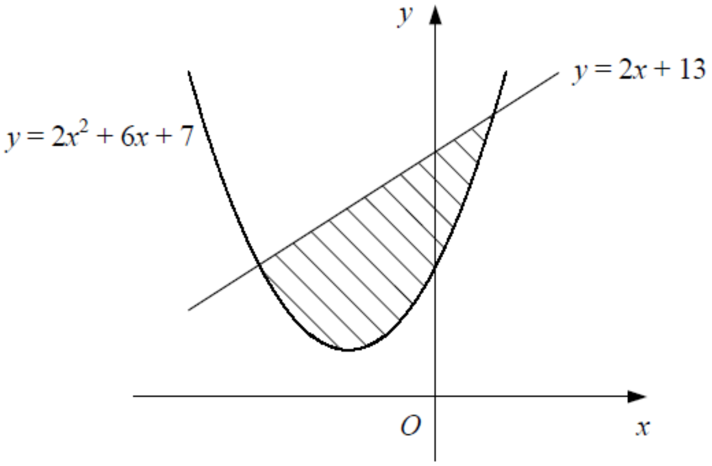


Figure 2

Figure 2 shows the curve $y = 2x^2 + 6x + 7$ and the straight line $y = 2x + 13$.

(a) Find the coordinates of the points where the curve and line intersect. **(4)**

(b) Find the area of the shaded region bounded by the curve and line. **(7)**

SECTION 2 – LOGS & EXPOENTIALS

EXERCISE 2A – USING BASIC LOG RULES & SOLVES

Solve exactly for *x*.

Need help?

1) $2\log_{10}x + \log_{10}\frac{1}{3} - \log_{10}\frac{3}{4} = 2$	2) $\log_x 3 + \log_x 27 = 2$
3) $\log_5 x = 16\log_x 5$	4) $\log_4(x+3) - \log_4 x = 3$
5) $\log_5(3x+95) = 2 + \log_5(x+3)$	6) $7^{x+1} = 3^{x+2}$

EXERCISE 2B – USING ln & e SOLVES

Solve exactly for x. Remember that lnx means $log_e x$ and the log rules apply in exactly the same way as any other log to a base

1) $3e^{2x} + 5 = 16e^{x}$	2) $(lnx)^2 + 2lnx - 15 = 0$
3) $e^{4x} - 3e^{2x} = -2$	4) $(lnx)^2 = 4(lnx + 3)$

EXERCISE 2C – EXAM PROBLEMS – LOG & EXPONENTIAL SOLVES

QUESTION 1

(a) Given that $t = \log_3 x$, find expressions in terms of t for

(i) $\log_3 x^2$,

(ii) $\log_9 x$. **(4)**

(b) Hence, or otherwise, find to 3 significant figures the value of x such that

$$\log_3 x^2 - \log_9 x = 4.$$ **(3)**

QUESTION 2

Giving your answers to 2 decimal places, solve the simultaneous equations

$$e^{2y} - x + 2 = 0$$

$$\ln(x + 3) - 2y - 1 = 0$$ **(8)**

SECTION 3 – GRAPH SKETCHING

You should know the standard curves $y = \frac{1}{x}, y = \frac{1}{x^2}, y = x^2, y = x^3, y = x^4, y = sinx, y = cosx,$ $y = tanx, y = secx, y = cosecx, y = cotx$. You will practice sketching these transformed curves below.

EXERCISE 3A – CURVE SKETCHING

Sketch the following curves, indicating the equations of any asymptotes or intersections with the coordinate axis.

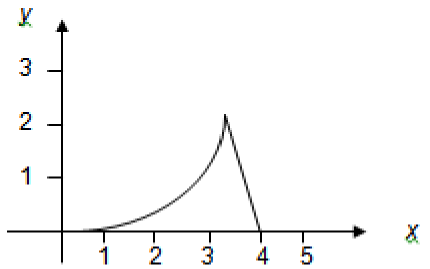
Need help? TRANSFORMATIONS 1 TRANSFORMATIONS 2

1) $y = \frac{1}{x-3}$	2) $y = 4^x - 2$
3) $y = -\log_3 x$	4) $y = \sin 3x \quad 0 \le x \le 2\pi$
5) $y = \frac{1}{2}e^x$	6) $y = \tan(-x) \quad -\pi \le x \le \pi$

7) $y = \sec x + 1 \quad 0 \le x \le 2\pi$	8) $y = \operatorname{cosec}\left(x - \frac{\pi}{2}\right) \quad -\pi < x < \pi$

EXERCISE 3B – TRANSFORMING ABSTRACT GRAPHS

The diagram shows the curve with the equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = 0$ for $x < 0$ or $x > 4$



Sketch the following:

1) $y = \mathrm{f}(x) - 1$	2) $y = \frac{1}{2}\mathrm{f}(x)$
3) $y = \mathrm{f}(2x)$	4) $y = \mathrm{f}(x+1)$

EXERCISE 3C – EXAM PROBLEMS – GRAPH TRANSFORMATIONS

QUESTION 1

(a) Describe fully a single transformation that maps the graph of $y = \frac{1}{x}$ onto the graph of $y = \frac{3}{x}$. **(2)**

(b) Sketch the graph of $y = \frac{3}{x}$ and write down the equations of any asymptotes. **(3)**

(c) Find the values of the constant c for which the straight line $y = c - 3x$ is a tangent to the curve $y = \frac{3}{x}$. **(4)**

QUESTION 2

$$f(x) = 4x - 3x^2 - x^3.$$

(a) Fully factorise $4x - 3x^2 - x^3$. **(3)**

(b) Sketch the curve $y = f(x)$, showing the coordinates of any points of intersection with the coordinate axes. **(3)**

QUESTION 3

The curve C has the equation

$$y = x^2 + ax + b,$$

where a and b are constants.

Given that the minimum point of C has coordinates $(-2, 5)$, find the values of a and b. **(4)**

SECTION 4 – TRIGONOMETRY

Trigonometry is a massive part of your A level. If the basics are solid then the more complicated questions will be much more manageable. The most important thing is knowing your identities as there are a lot of them and your ability to problem solve will be far stronger if you can recite the identities at the click of your fingers.

We will go step by step

EXERCISE 4A – BASIC TRIG SOLVES INCLUDING RECIPROCAL FUNCTIONS

To get your basics back up to par. These questions do not require any compound or double angle formulae. Solve the following for $0 \leq \theta \leq 2\pi$

Identities needed:

$\sin^2 x + \cos^2 x \equiv 1$

$tanx \equiv \frac{sinx}{cosx}$

$secx \equiv \frac{1}{cosx}, \quad cosecx \equiv \frac{1}{sinx}, \quad cotx \equiv \frac{cosx}{sinx} \equiv \frac{1}{tanx}$

$\sec^2 x \equiv 1 + \tan^2 x$

$cosec^2 x \equiv 1 + \cot^2 x$

Need help? TRIG 1 TRIG 2

1) $\sin 2\theta = 1$	2) $\tan 2\theta = 1.5$
3) $\cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{2}$	4) $\tan\left(\theta - \frac{\pi}{6}\right) = \sqrt{3}$

5) $\cos^2 x = \frac{1}{4}$	6) $\tan^2 x = 1$
7) $3\tan x = \cos x$	8) $\sin^2 x + \sin x = 2$
9) $\sec x = 2$	10) $\cot x = \sqrt{3}$

11) $\sec^2\theta - \tan\theta = 1$ for $-\pi \le \theta \le \pi$	12) $\cot^2\theta - 3\operatorname{cosec}\theta + 3 = 0$ for $0 \le \theta \le \pi$

EXERCISE 4B – TRIG SOLVES USING THE COMPOUND & DOUBLE ANGLES

Now we will step up our trig solves a notch, putting in everything we have just done but with the addition of the formulae below. Make sure you know these formulae off by heart !

Additional identities needed:

$$\sin(A \pm B) \equiv \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) \equiv \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin 2A \equiv 2\sin A\cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$$

$$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$$

Need help? COMPOUND DOUBLE

Solve the following $0 \leq \theta \leq 2\pi^c$

1) $\sin\left(x+\frac{\pi}{3}\right)+\cos\left(x+\frac{\pi}{6}\right)=\frac{1}{2}$	2) $\sin 2\theta - \sin\theta = 0$
3) $2\sin 2\theta + \cos\theta = 0$	4) $4\cos 2\theta - 6\cos\theta + 5 = 0$

5) $\sin 2\theta = 2\cos 2\theta$	6) $\cos 2\theta + \sin\theta - 1 = 0$
7) $3\cos 2\theta - \cos\theta + 2 = 0$	8) $\tan 2\theta + \tan\theta = 0$

EXERCISE 4C – TRIG PROOFS USING ALL FORMULAE SO FAR

Now we are going to put everything we have just done together in the form of LHS = RHS proofs.

Need help? HOW TO SET UP A TRIG IDENTITY

Prove the following LHS=RHS problems

1) $(\sin\theta + \cos\theta)^2 - 2\sin\theta\cos\theta \equiv 1$	2) $\operatorname{cosec}\theta(1-\cos\theta)(1+\cos\theta) \equiv \sin\theta$
3) $\frac{1}{\sec^2\theta} + \frac{1}{\operatorname{cosec}^2\theta} \equiv 1$	4) $\operatorname{cosec}^2\theta + \sec^2\theta \equiv \operatorname{cosec}^2\theta\sec^2\theta$

5) $\frac{\sec\theta}{\tan\theta+\cot\theta} \equiv \sin\theta$

6) $\frac{\sin\theta}{1-\cos\theta} + \frac{\sin\theta}{1+\cos\theta} \equiv 2 \operatorname{cosec} \theta$

7) $\cos x \cos 2x + \sin x \sin 2x \equiv \cos x$

8) $\sin(A+B)\sin(A-B) \equiv \sin^2 A - \sin^2 B$

9) $\tan A + \cot A \equiv 2\operatorname{cosec} 2A$

10) $\cos 2A \equiv \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$

EXERCISE 4D – EXAM PROBLEMS - TRIG PROBLEMS USING ALL FORMULAE SO FAR

QUESTION 1

Giving your answers in terms of π, solve the equation

$$3\tan^2\theta - 1 = 0,$$

for θ in the interval $-\pi \leq \theta \leq \pi$. **(6)**

QUESTION 2

(a) *(i)* Show that

$$\sin (x + 30)^\circ + \sin (x - 30)^\circ \equiv a \sin x^\circ,$$

where a is a constant to be found.

(ii) Hence find the exact value of $\sin 75^\circ + \sin 15^\circ$, giving your answer in the form $b\sqrt{6}$. **(6)**

(b) Solve, for $0 \leq y \leq 360$, the equation

$$2 \cot^2 y^\circ + 5 \operatorname{cosec} y^\circ + \operatorname{cosec}^2 y^\circ = 0.$$ **(6)**

QUESTION 3

(a) Prove that, for $\cos x \neq 0$,

$$\sin 2x - \tan x \equiv \tan x \cos 2x.$$ **(5)**

(b) Hence, or otherwise, solve the equation

$$\sin 2x - \tan x = 2 \cos 2x,$$

for x in the interval $0 \leq x \leq 180°$. **(5)**

SECTION 5 – DIFFERENTIATION

Like trigonometry, calculus (differentiation & integration) will form a massive part of your A level. If your differentiation is strong, it's likely your integration will be strong too as you will <u>always</u> be differentiating to check! You should know the differentiation table below off by heart. If you don't already, then write it out over and over for 10 minutes solid – trust us, you will learn that table in 10 minutes, and it will be extremely valuable to you when problem solving.

The differentiation table:

$$\frac{d}{dx}(sinx) = cosx$$

$$\frac{d}{dx}(cosx) = -sinx$$

$$\frac{d}{dx}(tanx) = \sec^2 x$$

$$\frac{d}{dx}(secx) = secxtanx$$

$$\frac{d}{dx}(cosecx) = -cosecxcotx$$

$$\frac{d}{dx}(cotx) = -cosec^2x$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)e^{f(x)}$$

$$\frac{d}{dx}(lnx) = \frac{1}{x}$$

$$\frac{d}{dx}(\ln f(x)) = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx}(a^x) = a^x lna$$

$$\frac{d}{dx}\left(a^{f(x)}\right) = f'(x)a^{f(x)} lna$$

EXERCISE 5A – THE CHAIN RULE

Need help?

Find $\frac{dy}{dx}$

1) $y=\left(x^2-3\right)^2$	2) $y=\left(3-x\right)^{10}$
3) $y=\frac{1}{\left(5x+2\right)^3}$	4) $y=\frac{10}{\left(1-x\right)^4}$
5) $y=e^{5x^3-2x^2+x-2}$	6) $y=e^{sinx}$

7) $y = \tan(3x + 1)$	8) $y = -\sec(4x^2 + x)$
9) $y = -4\cot\left(\frac{2x}{3}\right)$	10) $y = \sin^3 x$
11) $y = x + \sqrt{\sin x}$	12) $y = (\sin 2x)^{\frac{3}{2}}$

13) $y = cosec^3x$	14) $y = -4\tan^{\frac{1}{2}}(5x+1)$
15) $y = \ln(5x^3 + 2x^2 + x - 2)$	16) $y = \ln(secx)$
17) $y = 3^x$	18) $y = 4^{tanx}$

EXERCISE 5B – THE PRODUCT RULE & QUOTIENT RULES

Now that your chain rule is really strong you can build on this further whilst using the Product & Quotient Rules. Remember – differentiating a fraction, use the Quotient Rule ! That's what it's there for!

Product Rule:

If $y = uv, \quad then \ \frac{dy}{dx} = uv' + vu'$

Quotient Rule:

If $y = \frac{u}{v} \ , \ then \frac{dy}{dx} = \frac{vu' - uv'}{v^2}$

Need help?

Find $\frac{dy}{dx}$

1) $y = (3x+1)^4(x-3)$	2) $y = x^3\sqrt{4x-1}$
3) $y = x^2 \sin x$	4) $y = secxcosec3x$

5) $y = e^{5x^2}\cot 3x$

6) $y = 3x^5\ln(2x+1)$

7) $y = \dfrac{x^3}{(x-3)^4}$

8) $y = \dfrac{x}{\sqrt{(4x+3)}}$

9)

$y = \dfrac{\sin 2x}{x^2}$

10)

$y = \dfrac{\cos x}{\sin x}$

EXERCISE 5C – EXAM PROBLEMS – CHAIN, PRODUCT & QUOTIENT RULES

QUESTION 1

Given that

$$x = \sec^2 y + \tan y,$$

show that

$$\frac{dy}{dx} = \frac{\cos^2 y}{2\tan y + 1}. \qquad \textbf{(4)}$$

QUESTION 2

Differentiate each of the following with respect to x and simplify your answers.

(a) $\ln(3x - 2)$ **(2)**

(b) $\dfrac{2x+1}{1-x}$ **(3)**

(c) $x^{\frac{3}{2}} e^{2x}$ **(3)**

QUESTION 3

(a) Use the derivatives of $\sin x$ and $\cos x$ to prove that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\tan x) = \sec^2 x. \qquad \textbf{(4)}$$

The tangent to the curve $y = 2x \tan x$ at the point where $x = \frac{\pi}{4}$ meets the y-axis at the point P.

(b) Find the y-coordinate of P in the form $k\pi^2$ where k is a rational constant. **(6)**

SECTION 6 – INTEGRATION

As said before, if your differentiation is strong, then your integration has a good chance at being strong too. You will need to know both your differentiation and your trig identities really well to integrate fluently. We will work through these integrals step by step: standard integrals (moving backwards from the differentiation table), reverse chain rule, and then using trig identities to manipulate the integral to look like the right hand side of the differentiation table, rendering it a standard or reverse chain integral problem.

THE most important discipline that you need to adopt is to constantly DIFFERENTIATE TO CHECK immediately after you think you have found your answer.

EXERCISE 6A – STANDARD INTEGRALS & REVERSE CHAIN RULE

Need help?

Evaluate the following integrals

1) $\int 5e^{x} - 4sinx + 2x^{3}dx$	2) $\int 3secxtanx - \frac{2}{x}dx$
3) $\int \frac{1}{2x} + 2cosec^{2}xdx$	4) $\int 2cosecxcotx - \sec^{2} x\, dx$

5) $\int x(3x^2-5)^6dx$	6) $\int 3x(4x^2-5)^8dx$
7) $\int \frac{4x}{(2x^2-7)^5}dx$	8) $\int -\frac{6x^3}{(1-5x^4)^3}dx$
9) $\int \cos(3x+2)sin^4(3x+2)dx$	10) $\int 5sin2xcos^9 2xdx$

11) $\int -7\sec^2 9x \tan^4 9x\, dx$	12) $\int 3cosec^6 8x cot 8x dx$
13) $\int \frac{2\sec^2 2x}{(1+\tan 2x)^3} dx$	14) $\int 4xe^{8x^2-3} dx$
15) $\int \frac{4x+6}{x^2+3x} dx$	16) $\int \frac{\sec^2 2x}{(1+tan 2x)} dx$

EXERCISE 6B – INTEGRATION USING TRIG IDENTITIES
We will now merge everything we have just done together, and go one step further by incorporating all of the trig formulae we learnt from before into solving integrals that require a trig identity conversion.

Two further identities to learn and use – if you see an integral asking you to integrate a $\sin^2 ANGLE \;\; or\ a \cos^2 ANGLE$ you will need to exploit the double angle formulae and convert them to:

$\cos^2 x = \frac{1}{2} + \frac{1}{2} cos2x$ -> or $\cos^2 ANGLE = \frac{1}{2} + \frac{1}{2} cosDOUBLE\ THE\ ANGLE$

$\sin^2 x = \frac{1}{2} - \frac{1}{2} cos2x$ -> or $\sin^2 ANGLE = \frac{1}{2} - \frac{1}{2} cosDOUBLE\ THE\ ANGLE$

Need help?

Evaluate the following integrals

1) $\int (cotx - cosecx)^2 dx$	2) $\int \frac{cos2x}{\cos^2 x} dx$

3) $\int \frac{cos2x}{1-\cos^2 2x} dx$

4) $\int \sin^2 x \cos^2 x \, dx$ *hint: remember $sin2x = 2sinxcosx$

5) $\int -6 \sin^2 7x \, dx$

6) $\int 10 \cos^2 2x \, dx$

EXERCISE 6C – PARTIAL FRACTIONS & INTEGRATING PARTIAL

Remember that Partial Fractions is the last method you would use when approaching an integral in fraction form.

Order of thought when $\int FRACTION$:

- Is it $nf(x)$?
- If I bring the denominator up, so the power becomes negative, do I have a product in reverse chain rule form?
- Can I factorise the expression and cancel it down?
- Can I long divide it first, and then use ln or reverse chain?
- Partial Fractions

Need help? *NOTE – if sound doesn't for this QR doesn't work, type in this URL manually: https://www.youtube.com/watch?v=bjukJ7Q30Fk&feature=youtu.be

Evaluate the following integrals using Partial Fractions

1) $\int \frac{2x+11}{(x+1)(x+4)} dx$	2) $\int \frac{2x^2+2x-18}{x^3-6x^2+9x} dx$

3) $\int \frac{x^2+5x+7}{(x+2)^3}\,dx$	4) $\int \frac{4x^2+17x-11}{x^2+3x-4}\,dx$

EXERCISE 6D – EXAM PROBLEMS – REVERSE CHAIN & TRIG IDENTITY INTEGRALS

QUESTION 1

$$f(x) = \frac{15-17x}{(2+x)(1-3x)^2}, \quad x \neq -2, \quad x \neq \tfrac{1}{3}.$$

(a) Find the values of the constants A, B and C such that

$$f(x) = \frac{A}{2+x} + \frac{B}{1-3x} + \frac{C}{(1-3x)^2}.$$ **(4)**

(b) Find the value of

$$\int_{-1}^{0} f(x)\ dx,$$

giving your answer in the form $p + \ln q$, where p and q are integers. **(7)**

QUESTION 2

Find

$$\int \cot^2 2x \, dx. \qquad \textbf{(4)}$$

QUESTION 3

(a) Find the values of the constants A, B, C and D such that

$$\frac{2x^3 - 5x^2 + 6}{x^2 - 3x} \equiv Ax + B + \frac{C}{x} + \frac{D}{x-3}. \qquad \textbf{(5)}$$

(b) Evaluate

$$\int_1^2 \frac{2x^3 - 5x^2 + 6}{x^2 - 3x} \, dx,$$

giving your answer in the form $p + q \ln 2$, where p and q are integers. **(5)**

SECTION 7 – BINOMIAL EXPANSION

EXERCISE 7A – FINITE BINOMIAL EXPANSION

Formulae to know

Finite Binomial – When n is a positive integer:
$(a+bx)^n = ({}^nC_0)\,(a)^n(bx)^0 + ({}^nC_1)\,(a)^{n-1}(bx)^1 + ({}^nC_2)\,(a)^{n-2}(bx)^2 + \cdots$

Need help? INTRODUCTION VIDEO+EXAMPLE MORE EXAMPLES

Expand the following binomially in ascending powers of x:

1) $(3-4x)^5$	2) $(6-2x)^8$ $up\ to\ x^3$
3) $(a^2-b^2)^5$	4) $\left(\frac{1}{x}+x^2\right)^3$

EXERCISE 7B – INFINITE BINOMIAL EXPANSION

Formulae to know

Infinite Binomial – When n is not a positive integer:

$$(a+bx)^n = a^n\left(1+\frac{bx}{a}\right)^n = a^n[1+n\left(\frac{bx}{a}\right)+\frac{n(n-1)}{2!}\left(\frac{bx}{a}\right)^2+\frac{n(n-1)(n-2)}{3!}\left(\frac{bx}{a}\right)^3+\cdots$$

Need help?

Expand the following binomially in ascending powers of x up to x^3 :

1) $(2-x)^{-2}$

2) $\sqrt{9-2x}$

3) $\frac{1}{3-x}$

4) $\frac{1+2x}{(3+x)^2}$

QUESTION 1

(a) Expand $(1 + 3x)^8$ in ascending powers of x up to and including the term in x^3.
You should simplify each coefficient in your expansion. **(4)**

(b) Use your series, together with a suitable value of x which you should state, to estimate the value of $(1.003)^8$, giving your answer to 8 significant figures. **(3)**

QUESTION 2

$$f(x) = \frac{3}{\sqrt{1-x}}, \quad |x| < 1.$$

(a) Show that $f(\frac{1}{10}) = \sqrt{10}$. **(2)**

(b) Expand $f(x)$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(3)**

(c) Use your expansion to find an approximate value for $\sqrt{10}$, giving your answer to 8 significant figures. **(1)**

(d) Find, to 1 significant figure, the percentage error in your answer to part *(c)*. **(2)**

SECTION 8 – FUNCTIONS

EXERCISE 8A – DOMAINS & RANGES

Need help?

State the range of the following functions (you may need a sketch to help with this):

1) $f: x \to 3x + 2,\ domain\ x \geq 0$	2) $f(x) = x^2 + 5,\ domain\ x \geq 2$
3) $f: x \to 2sinx,\ domain\ 0 \leq x \leq 180^o$	4) $f: x \to \sqrt{x+2},\ domain\ x \geq -2$
5) $f(x) = e^x,\ domain\ x \geq 0$	6) $f(x) = 7logx,\ \ x \in R, x > 0$

EXERCISE 8B – COMPOSITE FUNCTIONS

Need help?

The functions $f\ and\ g$ are defined by $f(x) = 3x + 2,\ and\ g(x) = x^2 + 4.$ Find:

1) $fg(x)$	2) $gf(x)$
3) $f^2(x)$	4) $The\ values\ of\ b\ such\ that\ fg(b) = 62$

EXERCISE 8C – INVERSE FUNCTIONS

Need help?

For the following functions $g(x)$ with a restricted domain
i) state the domain and range of $g^{-1}(x)$
ii) the equation of the inverse function

1) $g(x) = \frac{1}{x},\ x \in R, x \geq 3$	2) $g(x) = 2x - 1,\ x \in R, x \geq 0$
3) $g(x) = \frac{3}{x-2},\ x \in R, x > 2$	4) $g(x) = \sqrt{x-3},\ x \in R, x \geq 7$

QUESTION 1

The functions f and g are defined by

$$\text{f} : x \rightarrow 3x - 4, \quad x \in \mathbb{R},$$

$$\text{g} : x \rightarrow \frac{2}{x+3}, \quad x \in \mathbb{R}, \quad x \neq -3.$$

(a) Evaluate fg(1). **(2)**

(b) Solve the equation gf(x) = 6. **(4)**

QUESTION 2

$$f(x) \equiv 2x^2 + 4x + 2, \quad x \in \mathbb{R}, \quad x \geq -1.$$

(a) Express $f(x)$ in the form $a(x+b)^2 + c$. **(2)**

(b) Describe fully two transformations that would map the graph of $y = x^2, \; x \geq 0$ onto the graph of $y = f(x)$. **(3)**

(c) Find an expression for $f^{-1}(x)$ and state its domain. **(4)**

(d) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram and state the relationship between them. **(4)**

SECTION 9 – STATISTICS – CODING

EXERCISE 9 – CODING – EXAM QUESTIONS

Need help? Read the below:

Coded data will always be in the form $Y = aX + b$
If you can rewrite the equation given to you in this form, then finding the mean and standard deviation of your coded data becomes very simple.

Mean of coded data:
$\bar{Y} = a\bar{X} + b$

Standard deviation of coded data:
$\sigma_y = a\sigma_x$

QUESTION 1
The weekly income, i, of 100 women workers was recorded.

The data coded using $y = \frac{i-90}{100}$ and the following summations were obtained:

$$\sum y = 131,\ \sum y^2 = 176.84$$

Estimate the standard deviation of the actual women workers' weekly income.

QUESTION 2
A meteorologist collected data on the annual rainfall, x mm, at six randomly selected places. The data was coded using $s = 0.01x - 10$ and the following summations were obtained:

$$\sum s = 16.1,\quad \sum s^2 = 147.03$$

Work out an estimate for the standard deviation of the actual annual rainfall.

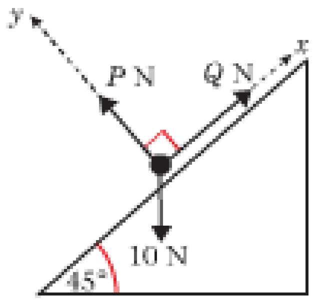
SECTION 10 – MECHANICS - SLOPES

EXERCISE 10A – STATIC PARTICLES ON SLOPES

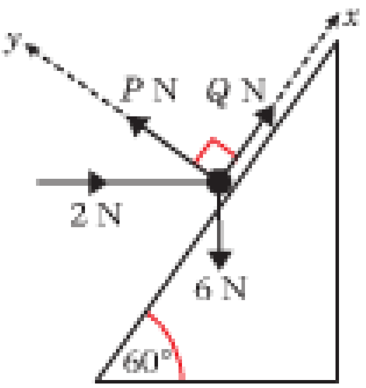
Need help? BASICS – HOW TO RESOLVE MORE + CONNECTED PARTICLES

Each of the particles are held in equilibrium. Find the magnitudes of P and Q:

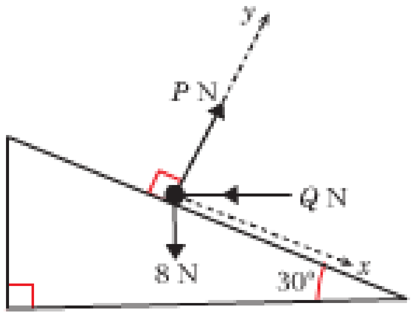
1)



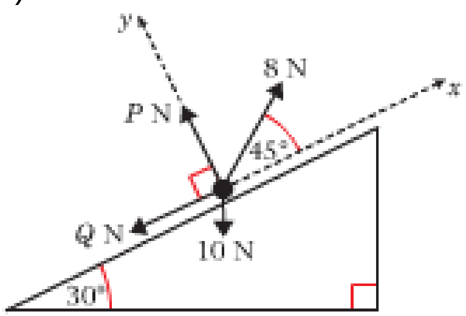
2)



3)



4)



EXERCISE 10B – ACCELERATING PARTICLES ON SLOPES

1)
A particle is held at rest on a rough plane which is inclined to the horizontal at an angle α, where $\tan \alpha = 0.75$. The coefficient of friction between the particle and the plane is 0.5. The particle is released and slides down the plane. Find

a the acceleration of the particle,

b the distance it slides in the first 2 seconds.

2)
A box of mass 2 kg is pushed up a rough plane by a horizontal force of magnitude 25 N. The plane is inclined to the horizontal at an angle of 10°. Given that the coefficient of friction between the box and the plane is 0.3, find the acceleration of the box.