BHASVIC MɑTHS

Doubles Tracking Test 1A

(45 minutes) – 38 marks

Name______________________________

1. Given that for all values of *x*: $3x^2 + 12x + 5 = p(x + q)^2 + r$

   (a) Find the values of *p*, *q* and *r*. (3 marks)

   (b) Hence solve the equation $3x^2 + 12x + 5 = 0$. (2 marks)

2. The curve C has the equation

$$y = x^3 - 9x$$

(a) Sketch the graph of C. (2 marks)

(b) Hence sketch on a separate diagram the graph of

$$y = (x+2)^3 - 9(x+2)$$ (2 marks)

Each of the two sketches must include the coordinates of all the points where the curve meets the coordinate axes.

3. Prove, from first principles, that the derivative of $4x^2$ is $8x$

(4 marks)

4.

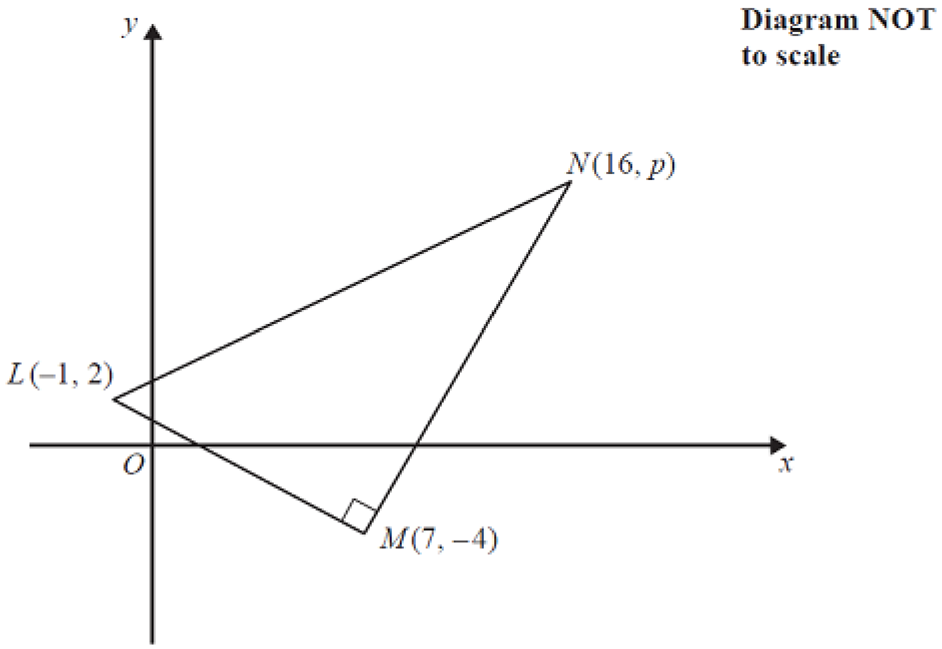

Figure 1

Figure 1 shows a right angled triangle *LMN*.

The points *L* and *M* have coordinates (–1, 2) and (7, –4) respectively.

(*a*) Find an equation for the straight line passing through the points *L* and *M*.

Give your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers.

(4 marks)

Given that the coordinates of point *N* are (16, *p*), where *p* is a constant, and angle $LMN = 90°$,

(*b*) find the value of *p*.

(3 marks)

Given that there is a point *K* such that the points *L*, *M*, *N*, and *K* form a rectangle,

(*c*) find the *y* coordinate of *K*.

(2 marks)

5.

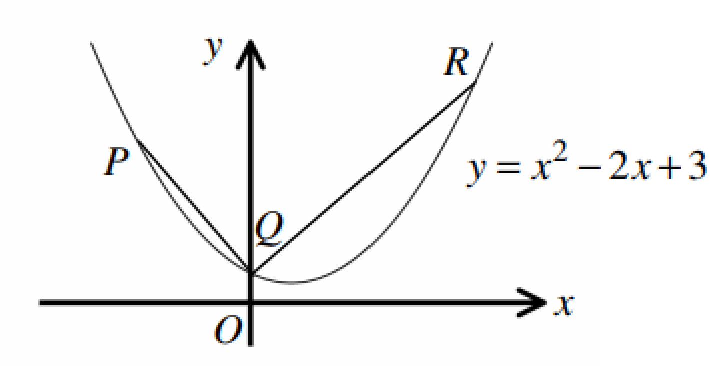

Figure 2

Figure 2 shows the curve, *C*, with the equation

$$y = x^2 - 2x + 3$$

The points *P* (-1 , 6), *Q* (0 , 3) and *R* all lie on *C*.

Given that angle PQR = 90°, determine the exact coordinates of *R*.

(6 marks)

6. The curve C has the equation

$$y = \frac{(x^2+4)(x-3)}{2x} \qquad x \neq 0.$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in its simplest form.

(5 marks)

(b) Find an equation of the tangent to C at the point where $x = -1$

Give your answer in the form $ax + by + c = 0$, where a, b, and c are integers.

(5 marks)

TT1 Part A

Answer:

1: (a) p=3, q=2, r=-7 — M1 for correct attempt at completing the square or comparing coefficients

A1 for 3, 2 A1 for -7

(a) $x + 2 = \pm\sqrt{\frac{7}{3}}$ — M1 must have plus/minus

$-2 \pm \sqrt{\frac{7}{3}}$ — A1 for correct answer

2: (a) M1 Correct Shape

A1 x intercept at $\pm$ 3, 0 (also y intercept = 0)

(b) M1 Horizontal translation
A1 Correct direction and all intercepts.

3:

$$\mathrm{f}(x) = 4x^2$$
$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$
$$= \lim_{h \to 0} \frac{4(x+h)^2 - 4x^2}{h}$$
$$= \lim_{h \to 0} \frac{4x^2 + 8xh + 4h^2 - 4x^2}{h}$$
$$= \lim_{h \to 0} \frac{8xh + 4h^2}{h}$$
$$= \lim_{h \to 0} \frac{h(8x + 4h)}{h}$$
$$= \lim_{h \to 0} (8x + 4h)$$

As $h \to 0$, $8x + 4h \to 8x$.
So $\mathrm{f}'(x) = 8x$

M1 Attempt at correct formula

A1 Correct expansion

M1 Remove factor of *h*

A1 Full proof.

4.(a)	Method 1: $gradient = \frac{y_1 - y_2}{x_1 - x_2} = \frac{2-(-4)}{-1-7}, = -\frac{3}{4}$	Method 2: $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$, so $\frac{y - y_1}{6} = \frac{x - x_1}{-8}$	M1, A1
	$y - 2 = -\frac{3}{4}(x+1)$ or $y + 4 = -\frac{3}{4}(x-7)$ or $y = their\ '-\frac{3}{4}'x + c$		M1
	$\Rightarrow \pm(4y + 3x - 5) = 0$		A1 (4)
	Method 3: Substitute *x* = -1, *y* = 2 and *x* = 7, *y* = -4 into *ax + by + c=0*		M1
	-*a* +2*b* + *c* = 0 and 7*a* - 4*b* + *c* = 0		A1
	Solve to obtain *a* = 3, *b* = 4 and *c* = -5 or multiple of these numbers		M1 A1 (4)
(b)	Attempts $gradient\ LM \times gradient\ MN = -1$ so $-\frac{3}{4} \times \frac{p+4}{16-7} = -1$ or $\frac{p+4}{16-7} = \frac{4}{3}$	Or $(y+4) = \frac{4}{3}(x-7)$ equation with *x* = 16 substituted	M1
	$p + 4 = \frac{9 \times 4}{3} \Rightarrow p = \ldots$, $p = 8$	So *y* =, *y* = 8	M1, A1
			(3)
(c)	Either (*y*=) $p+6$ or 2 + *p* + 4	Or use 2 perpendicular line equations through L and *N* and solve for *y*	M1
	(*y* =) 14		A1 (2)
			(9 marks)

5: M1 Method to find gradient PQ to find Gradient QR

M1 Method to find equation of QR

A1 $y = \frac{1}{3}x + 3$ (any form okay)

M1 Attempt to solve simultaneously ft. quadratic solving method $\frac{1}{3}x + 3 = x^2 - 2x + 3$

A1 (both) $x = 0$ (point Q) $x = \frac{7}{3}$

at $x = \frac{7}{3}$, $y = \frac{34}{9}$

A1 $(\frac{7}{3}, \frac{34}{9})$

6. (a)	$(x^2+4)(x-3)=x^3-3x^2+4x-12$	M1
		M1
	$\dfrac{x^3-3x^2+4x-12}{2x}=\dfrac{x^2}{2}-\dfrac{3}{2}x+2-6x^{-1}$	
		A1
	$\dfrac{dy}{dx}=x-\dfrac{3}{2}+\dfrac{6}{x^2}$	ddM1
	oe e.g. $\dfrac{2x^3-3x^2+12}{2x^2}$	A1
		(5)
(b)	At $x=-1,\ y=10$	B1
	$\left(\dfrac{dy}{dx}=\right)-1-\dfrac{3}{2}+\dfrac{6}{1}=3.5$	M1A1
	$y-'10'='3.5'\left(x--1\right)$	M1
	$2y-7x-27=0$	A1
		(5)
		(10 marks)