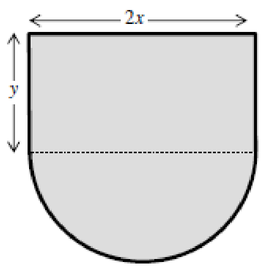
Doubles Tracking Test 2 part A

(38 marks: 46 minutes)

Name: ____________________

Teacher: _________________

1)



The figure below shows the design of a theatre stage which is the shape of a semicircle attached to a rectangle. The diameter of the semicircle is $2x$ m and is attached to one side of the rectangle also measuring $2x$ m. The other side of the rectangle is y m.

a) Given that the perimeter of the stage is 60 m. Show that the total area of the stage, $A\ m^2$ is given by

$$A = 60x - 2x^2 - \frac{1}{2}\pi x^2$$

(5 marks)

b) Find the maximum area of the stage to 3 s.f., and justify why this is a maximum area.

(6 marks)

2)

a) Show that

$$3\sec^2 x - 4\tan x \equiv \sec^2 x\,(3 - 4\sin x\cos x)$$

(3 marks)

b) Hence, find all the solutions to the equation $7 = \sec^2 x(3 - 4\sin x\cos x)$ in the interval $-180° \leq x \leq 180°$, giving your answers to 1 decimal place.

[*solutions based entirely on graphical or numerical methods are not acceptable*]

(6 marks)

3) The quadratic curves with equations

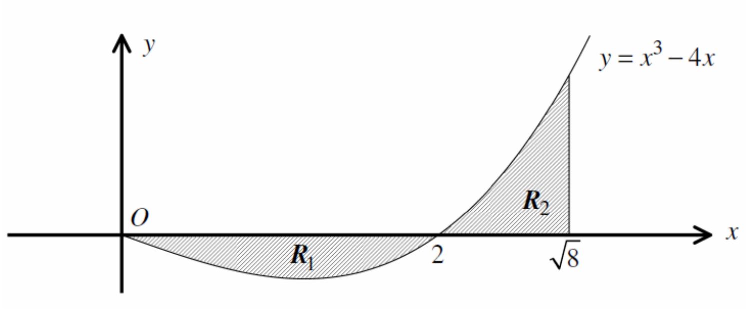
$$y = k\left(2x^2 + 1\right) \quad \text{and} \quad y = x^2 - 2x,$$

where k is a constant, **touch** each other.

Determine the possible values of k. Give your answer in set notation.

(5 marks)

4)



The figure above shows the cubic curve with equation

$$y = x^3 - 4x, \ x \geq 0.$$

The curve meets the x axis at the origin O and at the point where $x = 2$.

The finite region R_1 is bounded by the curve and the x axis, for $0 \leq x \leq 2$.

The region R_2 is bounded by the curve and the x axis, for $2 \leq x \leq \sqrt{8}$.

Show that the area of R_1 is equal to the area of R_2.

(5 marks)

5) The cubic equation C passes through the origin O and its gradient function is

$$\frac{dy}{dx} = 6x^2 - 6x - 20 .$$

a) Show clearly that the equation of C can be written as

$$y = x(2x + a)(x + b),$$

where a and b are constants.

(6 marks)

b) Sketch the graph of C, indicating clearly the coordinates of the points where the graph meets the coordinate axes.

(2 marks)

End of Part A

Analysis Sheet

(to be filled in after paper has been marked)

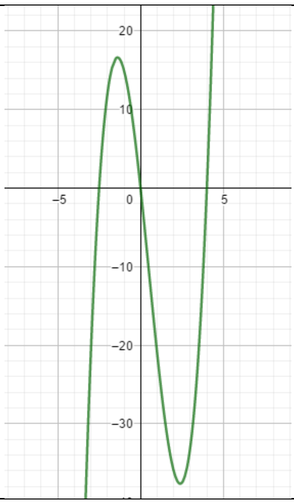
Part A							
Question	Topic	My mark	out of	%	Grade*	Action questions for the textbook	Submitted to teacher by when
1.			11			Pure 1 p281 Pure 2 p121	
2.			9			Pure 2 p157	
3.			5			Pure 1 p126	
4.			5			Pure 1 p304	
5.			8			Pure 1 p294	
TOTAL			38				
Part B							
1.			12			S and M 1 Ex 5B p74	
2.			7			Pure Book 1 p318	
3.			11			S and M 2 Ex 6C p 117	
4.			7			S and M 2 Ex 7A p130	
TOTAL			37				
Target grade				Overall Grade			

* (A* 90+, A 80+, B 70+, C 60+, D 50+, E 40% , less than 40 U (unclassified))

*Do 2 or 3 questions from each of these chapters in the books for questions where you achieved < 70% and bring in by the agreed date.

Mark Scheme

1a	$60 = 2x + 2x + \pi x$ $A = 2xy + \frac{\pi x^2}{2}$ $A = 60x - 2x^2 - \frac{\pi x^2}{2}$	M1 M1A1 M1A1
1b	$\frac{dA}{dx} = 60 - 4x - \pi x$ $0 = 60 - 4x - \pi x$ $x = \frac{60}{4+\pi}$	M1 M1 set to 0 and rearrange A1
1c	$A = 252$ $\frac{d^2A}{dx^2} = -4 - \pi$ $\therefore \frac{d^2A}{dx^2} < 0$ $\therefore A = 252\ is\ max$	B1 M1 A1
2a	$3\sec^2 x - 4\tan x \equiv \sec^2 x\,(3 - 4\sin x\cos x)$ $RHS = \frac{3 - 4\sin x \cos x}{\cos^2 x}$ $= 3\sec^2 x - \frac{4\sin x\cos x}{\cos^2 x}$ $= 3\sec^2 x - \frac{4\sin x}{\cos x}$ $= 3\sec^2 x - 4\tan x$	M1 ($\sec^2 x = \frac{1}{\cos^2 x}$ M1$\frac{\sin x}{\cos x} = \tan x$ A1 full solution, correct notation etc.
2b	$3(\tan^2 x + 1) - 7 - 4\tan x = 0$ $3\tan^2 x - 4\tan x - 4 = 0$ $(3\tan x + 2)(\tan x - 2) = 0$ $\tan x = 2$ $\tan x = -\frac{2}{3}$ $x = 63.4, -116.6, -33.7, 146.3$	M1 – subbing in A1 - -correct quadratic M1 – solving quaddratice M1 – a principle solution and ± 180 soltuion A1 2 correct solutions A1 All correct solutions
3	$k(2x^2 + 1) = x^2 - 2x$ $(2k-1)x^2 + 2x + k = 0$ $(-2)^2 - 4k(2k-1) = 0$ $2k^2 - k - 1 = 0$ $(2k+1)(k-1) = 0$ $k = -\frac{1}{2}, k = 1$	M1 A1 M1 M1 A1
4	$-R_1 = \int_0^2 x^3 - 4x\,dx$ $= \left(\frac{x^4}{4} - 2x^2\right)_0^2$ $= (4 - 8) = -4$ Area is below the curve therefore area=4 $R_2 = \int_2^{\sqrt{8}} x^3 - 4x\,dx$	M1 Integrating something M1 Subbing 2 in A1 Justification of 4 being the area

	$= \left(\frac{x^4}{4} - 2x^2\right)_2^{\sqrt{8}}$ $= (16 - 16) - (4 - 8)$ $= 4$ $R_1 = R_2$ Therefore areas are the same	M1 Subbing two sets of numbers and taking away A1
5a	$\frac{dy}{dx} = 6x^2 - 6x - 20$ $y = 2x^3 - 3x^2 - 20x + c$ $0 = c$ $y = x(2x + 5)(x - 4)$	M1A1 M1A1 M1A1
5b		B1Shape B1 roots