READING WEEK WORK

A. DOUBLES

# Exam-style practice

## Mathematics
## AS Level
## Paper 1: Pure Mathematics

**Time: 2 hours**
**You must have: Mathematical Formulae and Statistical Tables, Calculator**

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**1** a Given that $4 = 64^n$, find the value of $n$. **(1)**

b Write $\sqrt{50}$ in the form $k\sqrt{2}$ where $k$ is an integer to be determined. **(1)**

**2** Find the equation of the line parallel to $2x - 3y + 4 = 0$ that passes through the point (5, 6). Give your answer in the form $y = ax + b$ where $a$ and $b$ are rational numbers. **(3)**

**3** A student is asked to evaluate the integral $\int_1^2 \left(x^4 - \frac{3}{\sqrt{x}} + 2\right) dx$

The student's working is shown below

$$\int_1^2 \left(x^4 - \frac{3}{\sqrt{x}} + 2\right)dx = \int_1^2 (x^4 - 3x^{\frac{1}{2}} + 2)dx$$
$$= \left[\frac{x^5}{5} - 2x^{\frac{3}{2}} + 2x\right]_1^2$$
$$= \left(\frac{1}{5} - 2 + 2\right) - \left(\frac{32}{5} - 2\sqrt{8} + 4\right)$$
$$= -4.54 \text{ (3 s.f.)}$$

a Identify two errors made by the student. **(2)**

b Evaluate the definite integral, giving your answer correct to 3 significant figures. **(2)**

**4** Find all the solutions in the interval $0 \leqslant x \leqslant 180°$ of

$$2\sin^2(2x) - \cos(2x) - 1 = 0$$

giving each solution in degrees. **(7)**



**5** A rectangular box has sides measuring $x$ cm, $x + 3$ cm and $2x$ cm.

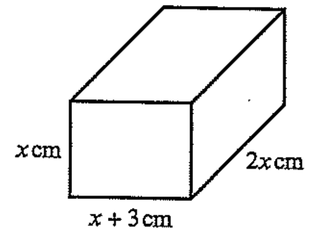



**Figure 1**

a Write down an expression for the volume of the box. **(1)**

Given that the volume of the box is 980 cm³,

b Show that $x^3 + 3x^2 - 490 = 0$. **(2)**

c Show that $x = 7$ is a solution to this equation. **(1)**

d Prove that the equation has no other real solutions. **(4)**

**6** $f(x) = x^3 - 5x^2 - 2 + \frac{1}{x^2}$

The point $P$ with $x$-coordinate $-1$ lies on the curve $y = f(x)$. Find the equation of the normal to the curve at $P$, giving your answer in the form $ax + by + c = 0$ where $a$, $b$ and $c$ are positive integers. **(7)**

**7** The population, $P$, of a colony of endangered Caledonian owlet-nightjars can be modelled by the equation $P = ab^t$ where $a$ and $b$ are constants and $t$ is the time, in months, since the population was first recorded.

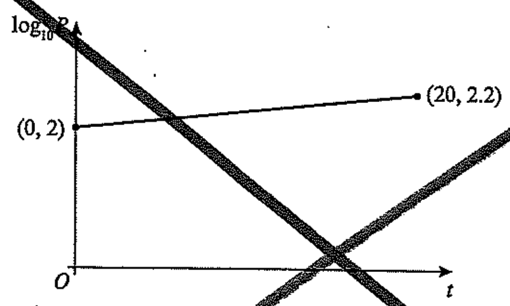



**Figure 2**

The line $l$ shown in figure 2 shows the relationship between $t$ and $\log_{10}P$ for the population over a period of 20 years.

a Write down an equation of line $l$. **(3)**

b Work out the value of $a$ and interpret this value in the context of the model. **(3)**

c Work out the value of $b$, giving your answer correct to 3 decimal places. **(2)**

d Find the population predicted by the model when $t = 30$. **(1)**

**8** Prove that $1 + \cos^4 x - \sin^4 x \equiv 2\cos^2 x$. **(4)**

**9** Relative to a fixed origin, point $A$ has position vector $6\mathbf{i} - 3\mathbf{j}$ and point $B$ has position vector $4\mathbf{i} + 2\mathbf{j}$.

Find the magnitude of the vector $\overrightarrow{AB}$ and the angle it makes with the unit vector $\mathbf{i}$. **(5)**

**10** A triangular lawn $ABC$ is shown in figure 3:

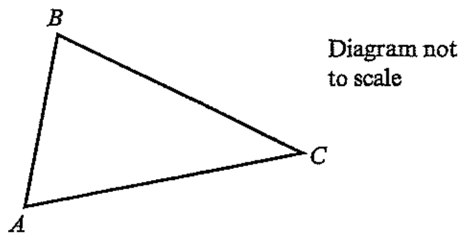



**Figure 3**

Given that $AB = 7.5\,\text{m}$, $BC = 10.6\,\text{m}$ and $AC = 12.7\,\text{m}$,

**a** Find angle $BAC$. **(3)**

Grass seed costs £1.25 per square metre.

**b** Find the cost of seeding the whole lawn. **(5)**

**11** $g(x) = (x-2)^2(x+1)(x-7)$

**a** Sketch the curve $y = g(x)$, showing the coordinates of any points where the curve meets or cuts the coordinate axes. **(4)**

**b** Write down the roots of the equation $g(x+3) = 0$. **(1)**

**12** Given that $9^{2x} = 27^{x^2-5}$, find the possible values of $x$. **(6)**

**13** $f(x) = (1-3x)^5$

**a** Expand $f(x)$, in ascending powers of $x$, up to the term in $x^2$. Give each term in its simplest form. **(3)**

**b** Hence find an approximate value for $0.97^5$. **(2)**

**c** State, with a reason, whether your approximation is greater or smaller than the true value. **(2)**

**14** $f'(x) = \dfrac{\sqrt{x} - x^2 - 1}{x^2}$, $x > 0$

**a** Show that $f(x)$ can be written as $f(x) = -\dfrac{x^2 + 2\sqrt{x} - 1}{x} + c$ where $c$ is a constant. **(5)**

Given that $f(x)$ passes through the point $(3, -1)$,

**b** find the value of $c$. Give your answer in the form $p + q\sqrt{r}$ where $p$, $q$ and $r$ are rational numbers to be found. **(4)**

**15** A circle, $C$, has equation $x^2 + y^2 - 4x + 6y = 12$

**a** Show that the point $A(5, 1)$ lies on $C$ and find the centre and radius of the circle. **(5)**

**b** Find the equation of the tangent to $C$ at point $A$. Give your answer in the form $y = ax + b$ where $a$ and $b$ are rational numbers. **(4)**

**c** The curve $y = x^2 - 2$ intersects this tangent at points $P$ and $Q$. Given that $O$ is the origin, find, as a fraction in simplest form, the exact area of the triangle $POQ$. **(7)**

## Exam-style practice

**1** **a** $\frac{1}{3}$ **b** $5\sqrt{2}$

**2** $y = \frac{2}{3}x + \frac{8}{3}$

**3** **a** error 1: $= -\dfrac{3}{\sqrt{x}} = -3x^{-\frac{1}{2}}$, not $-3x^{\frac{1}{2}}$

error 2: $\left[\dfrac{x^5}{5} - 2x^{\frac{3}{2}} + 2x\right]_1^2 = \left(\dfrac{32}{5} - 2\sqrt{8} + 4\right) - \left(\dfrac{1}{5} - 2 + 2\right)$

not $\left(\dfrac{1}{5} - 2 + 2\right) - \left(\dfrac{32}{5} - 2\sqrt{8} + 4\right)$

**b** 5.71 (3 s.f.)

**4** $x = 30°, 90°, 150°$

**5** **a** $2x^2(x+3)$

**b** $2x^2(x+3) = 980 \Rightarrow 2x^3 + 6x^2 - 980 = 0$
$\Rightarrow x^3 + 3x^2 - 490 = 0$

**c** For $f(x) = x^3 + 3x^2 - 490 = 0$, $f(7) = 0$, so $x - 7$ is a factor of $f(x)$ and $x = 7$ is a solution.

**d** Equation becomes $(x-7)(x^2 + 10x + 70) = 0$
Quadratic has discriminant $10^2 - 4 \times 1 \times 70 = -180$
So the quadratic has no real roots, and the equation has no more real solutions.

**6** $x + 15y + 106 = 0$

**7** **a** $\log_{10}P = 0.01t + 2$

**b** 100, initial population

**c** 1.023

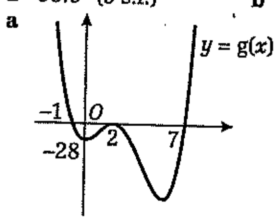
**d** Accept answers from 195 to 200

**8** $1 + \cos^4 x - \sin^4 x \equiv 1 + (\cos^2 x + \sin^2 x)(\cos^2 - \sin^2 x)$
$\equiv (1 - \sin^2 x) + \cos^2 x \equiv 2\cos^2 x$

**9** Magnitude $= \sqrt{29}$, angle $= 112°$ (3 s.f.)

**10** **a** 56.5° (3 s.f.) **b** £49.63

**11** **a**




**b** $-4, -1, 4$

**12** $x = 3$ or $-\frac{5}{3}$

**13** **a** $1 - 15x + 90x^2$

**b** 0.859

**c** Greater: The next term will be subtracting from this, and future positive terms will be smaller.

**14** **a** $f(x) = \int(x^{-\frac{3}{2}} - 1 - x^{-2})\,dx = -2x^{-\frac{1}{2}} - x + x^{-1} + c$

$= -\dfrac{x^2 + 2\sqrt{x} - 1}{x} + c$

**b** $c = \frac{5}{3} + \frac{2}{3}\sqrt{3}$

**15** **a** $5^2 + 1^2 - 4 \times 5 + 6 \times 1 = 12$, so $(5, 1)$ lies on $C$.
Centre $= (2, -3)$, radius $= 5$

**b** $y = -\frac{3}{4}x + \frac{19}{4}$

**c** $12\frac{15}{32}$