| $\alpha$ | $\beta$ | $\gamma$ | $\delta$ | $\varepsilon$ | $\zeta$ | $\eta$ | $\theta$ | $\iota$ | $\kappa$ | $\lambda$ | $\mu$ | $\nu$ | $\xi$ | $o$ | $\pi$ | $\rho$ | $\sigma$ | $\tau$ | $\upsilon$ | $\varphi$ | $\chi$ | $\psi$ | $\omega$ |
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*"Perhaps the greatest paradox of all is that there are paradoxes in mathematics"*

J Newman

# Further Maths A2 (M2FP2D1) Assignment $\iota$ (iota) A

**Due to be handed in after Reading Week w/b 20th Nov 17**

**PREPARATION** *You have an FP2 Mock exam 1½ hour paper the first week after half term in lesson time on all of FP2, so practice using the assignments.*

## CURRENT WORK

Solve the following first order linear differential equations of the type $\frac{\mathrm{d}y}{\mathrm{d}x} + Py = Q$, where $P$ and $Q$ are functions of $x$, by multiplying through the equation by an integrating factor to produce an exact equation.

1. $x^2 \frac{\mathrm{d}y}{\mathrm{d}x} - xy = \frac{x^3}{x+2} \quad x > -2$

2. Find $y$ in terms of $x$ given that $x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = e^x$ and that $y = 1$ when $x = 1$.

3. a) Find the general solution of the differential equation $\left(x + \frac{1}{x}\right)\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 2\left(x^2+1\right)^2$, given $y$ in terms of $x$.

   b) Find the particular solution which satisfies the condition that $y = 1$ at $x = 1$.

## CONSOLIDATION

4. By using

$$2\cos n\theta = \mathrm{e}^{\mathrm{i}n\theta} + \mathrm{e}^{-\mathrm{i}n\theta}$$

for suitable values of $n$, or otherwise, show that

$$2^5\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$$

Hence, or otherwise, evaluate

$$\int_0^{\frac{\pi}{4}} 2^5\cos^6\theta \, \mathrm{d}\theta$$

5. If $z = \cos\theta + \mathrm{i}\sin\theta$, show that

$$z - \frac{1}{z} = 2\mathrm{i}\sin\theta, \quad z^n - \frac{1}{z^n} = 2\mathrm{i}\sin n\theta$$

Hence, or otherwise, show that

$$16\sin^5\theta = \sin 5\theta - 5\sin 3\theta + 10\sin\theta$$

6. Find the modulus and argument of each of the complex numbers $z_1$ and $z_2$, where

$$z_1 = \frac{1+\mathrm{i}}{1-\mathrm{i}},\ z_2 = \frac{\sqrt{2}}{1-\mathrm{i}}$$

Plot the points representing $z_1$, $z_2$ and $z_1 + z_2$ on an Argand diagram. Deduce from your diagram that

$$\tan\left(\frac{3\pi}{8}\right) = 1 + \sqrt{2}$$

7. (a) Find the first four terms of the expansion, in ascending powers of *x*, of

$$(2x+3)^{-1}, |x| < \tfrac{2}{3}.$$

(b) Hence, or otherwise, find the first four non-zero terms of the expansion, in ascending powers of *x*, of $\dfrac{\sin 2x}{3+2x}, |x| < \tfrac{2}{3}$.

8. (a) Find the Taylor expansion of $\ln(\sin x)$ in ascending powers of $\left(x - \frac{\pi}{6}\right)$ up to and including the term in $\left(x - \frac{\pi}{6}\right)^3$.

(b) Use your answer to part **a** to obtain an estimate of $\ln(\sin 0.5)$, giving your answer to 6 decimal places.

9. Given that $y = \tan x$,

(a) Find $\dfrac{dy}{dx}, \dfrac{d^2y}{dx^2}$ and $\dfrac{d^3y}{dx^3}$.

(b) Find the Taylor series expansion of tan *x* in ascending powers of $\left(x - \frac{\pi}{4}\right)$ up to and including the term in $\left(x - \frac{\pi}{4}\right)^3$.

(c) Hence show that

$$\tan\frac{3\pi}{10} \approx 1 + \frac{\pi}{10} + \frac{\pi^2}{200} + \frac{\pi^3}{3000}.$$

10. Prove that

$$\left(z^n - e^{\mathrm{i}\theta}\right)\left(z^n - e^{-\mathrm{i}\theta}\right) = z^{2n} - 2z^n\cos\theta + 1$$

Hence, or otherwise, find the roots of the equation $z^6 - z^3\sqrt{2} + 1 = 0$
in the form $\cos\alpha + \mathrm{i}\sin\alpha$, $-\pi < \alpha \le \pi$

11. Show that in an Argand diagram the equation $\arg(z-2) - \arg(z-2\mathrm{i}) = \frac{3\pi}{4}$

Represents an arc of a circle and that $\left|\frac{z-4}{z-1}\right|$ is constant on this circle.

Find the values of $z$ corresponding to the points in which this circle is cut by the curve given by

$|z-1|+|z-4|=5$

12. Sketch on an Argand diagram the curve described by the equation $|z-3+6\mathrm{i}|=2|z|$ and express the equation of the curve in cartesian form.

13. Given that $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$

(a) show that the point $P$, which represents $z$ on an Argand diagram, lies on an arc of a circle.

(b) Find the centre and radius of this circle and sketch the locus of $P$.

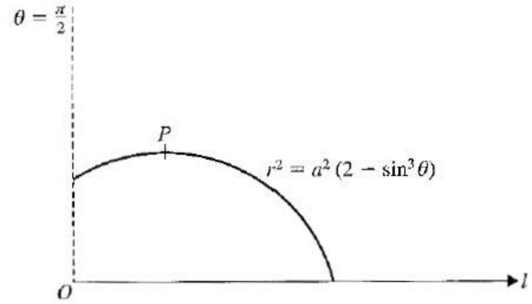
(c) Sketch, on a separate Argand diagram, the locus represented by $\left|\frac{z-3\mathrm{i}}{z+3}\right|=1$ .

14. The function f is defined for all real $x$ by $\mathrm{f}(x)=\begin{cases}\frac{1}{x} & \text{for} \quad x>0\\ |x| & \text{for} \quad x\le 0\end{cases}$

(a) Sketch the graph of f.

(b) Find the set of values of $x$ for which $\mathrm{f}(x)\le 4$.

15.




The diagram shows a sketch of the curve with equation $r^2=a^2\left(2-\sin^3\theta\right)$, $0\le\theta\le\frac{\pi}{2}$ where $a$ is a positive constant.

By considering the stationary values of $r^2\sin^2\theta$, or otherwise, find the polar coordinates of $P$, where $P$ is a point on the curve at which the tangent is parallel to the initial line $l$.

16. Obtain, in the form $r=\mathrm{f}(\theta)$, a polar equation of the curve whose Cartesian equation is

(a) $x+4y=3$

(b) $x^2+xy=2y$

**Answers:**

(1) $y = x\ln(x+2) + cx$

(2) $y = \frac{1}{x}e^x - \frac{1}{x^2}e^x + \frac{1}{x^2}$ (3a) $y = \frac{1}{3}(x^2+1)^2 + \frac{c}{(x^2+1)}$ (3b) $y = \frac{1}{3}(x^2+1)^2 - \frac{2}{3(x^2+1)}$

**Consolidation:**

(4) $\frac{22}{3} + \frac{5}{2}\pi$ (6) $z_1 \text{ mod}.1, \arg\frac{\pi}{2}$, $z_2 \text{ mod}.1, \arg\frac{\pi}{4}$ (7a) $\frac{1}{3} - \frac{2}{9}x + \frac{4}{27}x^2 - \frac{8}{81}x^3 + \ldots$

(7b) $\frac{2}{3} - \frac{4}{9}x^2 - \frac{4}{27}x^3 + \frac{8}{81}x^4 + \ldots$ (8a) $-\ln 2 + \sqrt{3}\left(x - \frac{\pi}{6}\right) - 2\left(x - \frac{\pi}{6}\right)^2 + \frac{4\sqrt{3}}{3}\left(x - \frac{\pi}{6}\right)^3 + \ldots$

(8b) –0.735166 (6 d.p.) (9a) $\frac{\mathrm{d}y}{\mathrm{d}x} = \sec^2 x$ (9b) $1 + 2\left(x - \frac{\pi}{4}\right) + 2\left(x - \frac{\pi}{4}\right)^2 + \frac{8}{3}\left(x - \frac{\pi}{4}\right)^3 + \ldots$

$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2\sec^2 x\tan x$

$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 4\sec^2 x\tan^2 x + 2\sec^4 x$

(10) $\cos\frac{\pi}{12} + \mathrm{i}\sin\frac{\pi}{12}$, $\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}$

$\cos\left(-\frac{7\pi}{12}\right) + \mathrm{i}\sin\left(-\frac{7\pi}{12}\right)$,

$\cos\left(-\frac{\pi}{12}\right) + \mathrm{i}\sin\left(-\frac{\pi}{12}\right)$

$\cos\left(-\frac{3\pi}{4}\right) + \mathrm{i}\sin\left(-\frac{3\pi}{4}\right)$, $\cos\frac{7\pi}{12} + \mathrm{i}\sin\frac{7\pi}{12}$

(11) $\frac{10}{9} \pm \frac{4\sqrt{14}}{9}\mathrm{i}$

(12) $(x+1)^2 + (y-2)^2 = 20$ (13b) centre (0, 1), radius $\sqrt{2}$

(14) $-4 \le x \le 0$ or $x \ge \frac{1}{4}$ (15) (1.095$a$, 1.19)

(16a) $r = \frac{3}{\cos\theta + 4\sin\theta}$ (16b) $r = \frac{2\tan\theta}{\cos\theta + \sin\theta}$

BHASVIC
MαTHS

# ASSIGNMENT COVER SHEET iota

Name ------------------------------------------------ Maths Teacher ------------------------------------------------

| Question | Done | Backpack | Ready for test | Notes |
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