α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	σ	τ	υ	φ	χ	ψ	ω

"The mathematician's patterns, like the painter's or the poet's, must be beautiful: the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test." G H Hardy

Further Maths A2 (M2FP2D1) Assignment θ (theta) A

Due 13th November Monday only or 20th November

DRILL

Drills are the very basic techniques you need to solve maths problems. You need to practise these until you can do them quickly and accurately. Answers are not provided for drill questions.

1. Express the following in the form $r(\cos\theta + \text{i}\sin\theta)$, where $-\pi < \theta \le \pi$. Give the exact values of r and θ where possible, or values to 2d.p. otherwise.

a) 7 b) -5i c) $\sqrt{3}+\text{i}$ d) 2 + 2i e) 1 - i

f) -8 g) 3 - 4i h) -8 + 6i i) $2-\sqrt{3}\text{i}$

2. Express the following in the form $x+\text{i}y$, where $x \in \mathbb{R}$ and $y \in \mathbb{R}$.

a) $5\left(\cos\frac{\pi}{2} + \text{i}\sin\frac{\pi}{2}\right)$ b) $\frac{1}{2}\left(\cos\frac{\pi}{6} + \text{i}\sin\frac{\pi}{6}\right)$

c) $6\left(\cos\frac{5\pi}{6} + \text{i}\sin\frac{5\pi}{6}\right)$ d) $3\left(\cos\left(-\frac{2\pi}{3}\right) + \text{i}\sin\left(-\frac{2\pi}{3}\right)\right)$

e) $2\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + \text{i}\sin\left(-\frac{\pi}{4}\right)\right)$ f) $-4\left(\cos\frac{7\pi}{6} + \text{i}\sin\frac{7\pi}{6}\right)$

PREPARATION

Every week you will be required to do some preparation for future lessons, to be advised by your teacher.

CURRENT WORK

1. (a) Given that $z = \cos\theta + \text{i}\sin\theta$, show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

where n is a positive integer.

(b) Hence show that $\cos^4\theta$ can be expressed in the form $p\cos 4\theta + q\cos 2\theta + r$, where p, q and r are constants, stating the values of p, q and r. [E]

2. (a) Use de Moivre's theorem to express $\cos 5\theta$ in terms of powers of $\cos\theta$.

(b) Solve the equation $z^3 = 8\text{i}$ giving each of the roots in the form $r\text{e}^{\text{i}\theta}$, where $r > 0$ and $0 \le \theta \le 2\pi$.

3. (a) If $z = 2e^{-i\frac{\pi}{3}}$, express z, z^2 and $\frac{1}{z}$ in the form $x + yi$ where x and y are real. State the modulus and argument of each of these complex numbers, giving the argument in each case as an angle θ such that $-\pi < \theta \leqslant \pi$. Represent the three numbers in an Argand diagram.

(b) By using de Moivre's theorem, or otherwise, show that

$$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$$

Solve the following first order linear differential equations of the type $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x, by multiplying through the equation by an integrating factor to produce an exact equation.

4. $\frac{dy}{dx} + 2y = e^x$

5. $\frac{dy}{dx} + y\sin x = e^{\cos x}$

6. $\frac{dy}{dx} + y\tan x = x\cos x$

7. By using the series expansions of e^x and $\cos x$, or otherwise, find the expansion of $e^x \cos 3x$ in ascending powers of x up to and including the term in x^3.

8. If $z = \cos\theta + i\sin\theta$, show that

$$z - \frac{1}{z} = 2i\sin\theta, \quad z^n - \frac{1}{z^n} = 2i\sin n\theta$$

Hence, or otherwise, show that $16\sin^5\theta = \sin 5\theta - 5\sin 3\theta + 10\sin\theta$

9. Given that $f(x) \equiv \ln\left(2x + \sqrt{(1+4x^2)}\right)$,

(a) show that $f'(x) \equiv \frac{2}{\sqrt{(1+4x^2)}}$

(b) Obtain the Maclaurin expansion for $f(x)$ in ascending powers of x, up to and including the term in x^3.
This series expansion is the same as the Maclaurin series expansion for sin (kx), up to and including the term in x^3.

(c)Write down the value of k.

10. Find the first three derivatives of $(1+x)^2 \ln(1+x)$. Hence, or otherwise, find the expansion of $(1+x)2\ln(1+x)$ in ascending powers of x up to and including the term in x^3.

11. $\frac{dy}{dx} = 2 + x + \sin y$ with $y = 0$ at $x = 0$

Use the Taylor series method to obtain y as a series in ascending powers of x up to and including the term in x^3, and hence obtain an approximate value for y at $x = 0.1$.

12. Prove, by induction or otherwise, that for any positive integer n,

$$n^3 + 6n^2 + 8n$$

is divisible by 3.

Answers:

(1b)	$p = \frac{1}{8},\ q = \frac{1}{2},\ r = \frac{3}{8}$	(2a)	$16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$
(2b)	$2e^{\frac{i\pi}{6}}, 2e^{\frac{i5\pi}{6}}, 2e^{-\frac{i\pi}{2}}$	(3a)	$1 - i\sqrt{3}, -2 - 2i\sqrt{3}, \frac{1}{4} + \frac{1\sqrt{3}}{4};$ $2, -\frac{\pi}{3}; 4, -\frac{2\pi}{3}, \frac{1}{2}, \frac{\pi}{3}$
(4)	$y = \frac{1}{3}e^x + ce^{-2x}$	(5)	$y = xe^{\cos x} + ce^{\cos x}$
(6)	$y = \left(\dfrac{x^2}{2} + c\right)\cos x$	(7)	$1 + x - 4x^2 - \frac{13}{3}x^3$
(9b)	$2x - \frac{3}{4}x^3 + \dots$	(9c)	$k = 2$
(10)	$2(1+x)\ln(1+x) + (1+x);\ \ 2\ln(1+x) + 3;\ \ \dfrac{2}{1+x}$		
(11)	$2x + \frac{3}{2}x^2 + \frac{1}{2}x^3;\ \ 0.2155$		

BHASVIC

MATHS

ASSIGNMENT COVER SHEET theta

Name -- Maths Teacher --

Question	Done	Backpack	Ready for test	Notes
Drill				
1				
2				
3				
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