α	β	γ	δ	ε	ζ	**η**	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	σ	τ	υ	φ	χ	ψ	ω

"Logic is the art of going wrong with confidence" M Kline

Further Maths A2 (M2FP2D1) Assignment η (eta) A

Due in 6th Nov

PREPARATION *Every week you will be required to do some preparation for future lessons, to be advised by your teacher.*

CURRENT WORK

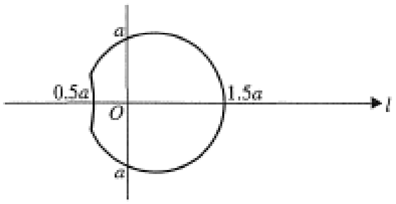
1. Find the set of values of *x* for which

$$\frac{x^2+7x+10}{x+1} > 2x+7$$

2. The curve shown has polar equation

$$r = a\left(1+\tfrac{1}{2}\cos\theta\right), a>0, 0<\theta \leqslant 2\pi$$

Determine the area enclosed by the curve, giving your answer in terms of *a* and π.



3. Sketch, on the same diagram, the curves C_1, C_2 whose polar equations are

$$C_1 : r = a(1+\cos\theta), -\pi < \theta \leqslant \pi$$
$$C_2 : r = b(1-\cos\theta), -\pi < \theta \leqslant \pi$$

where *a and b are positive constants with b > a*. Find the points common to C_1 and C_2 .
Calculate the area of the finite region bounded by the arcs with polar equations

$$r = a(1+\cos\theta),\ \tfrac{\pi}{2} \leqslant \theta \leqslant \pi$$
$$r = a(1-\cos\theta),\ 0 \leqslant \theta \leqslant \tfrac{\pi}{2}$$

4. Given that $x = At^2e^{-t}$ satisfies the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = e^{-t},$$

a) Find the value of *A*.

b) Hence find the solution of the differential equation for which *x* = 1 and $\frac{dx}{dt} = 0$ at *t* = 0.

c) Use your solution to prove that for $t \geq 0, x \leq 1$.

5. Find the solution subject to the given initial conditions for each of the following differential equations:

a) $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 2t - 3$ $\quad x = 2$ and $\frac{dx}{dt} = 4$ when $t = 0$

b) $\frac{d^2x}{dt^2} - 9x = 10\sin t$ $x = 2$ and $\frac{dx}{dt} = -1$ when $t = 0$

c) $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 4x = 3te^{2t}$ $x = 0$ and $\frac{dx}{dt} = 1$ when $t = 0$

6. Sketch the curve with equation $r = a(1+\cos\theta)$ for $0 \leqslant \theta \leqslant \pi$ where $a > 0$.

Sketch also the line with equation $r = 2a\sec\theta$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ on the same diagram.

The half-line with equation $\theta = \alpha, 0 < \alpha < \frac{\pi}{2}$, meets the curve at *A and the line equation* $r = 2a\sec\theta$ *at B. If O is the pole, find the value of* $\cos\alpha$ *for which OB = 2OA.*

7. Use the formula for the Maclaurin expansion and differentiation to show that

a) $(1-x)^{-1} = 1 + x + x^2 + \ldots + x^r + \ldots$ b) $\sqrt{(1+x)} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \ldots$

8. a) Show that the Maclaurin expansion for $\cos x$ is $1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots$

b) Using the first 3 terms of the series, show that it gives a value for cos 30° correct to 3 decimal places.

9. Use Maclaurin's expansion and differentiation to expand, in ascending powers of *x* up to and including the term in x^4,

a) e^{3x} b) $\ln(1+2x)$ c) $\sin^2 x$

10. Given that $f(x) = (1-x)^2 \ln(1-x)$

a) Show that $f''(x) = 3 + 2\ln(1-x)$

b) Find the values of $f(0), f'(0), f''(0), \text{and} f'''(0)$,

c) Express $(1-x)^2 \ln(1-x)$ in ascending powers of *x* up to and including the term in x^3.

11. Given that $f(x) = \ln\cos x$,

a) Show that $f'(x) = -\tan x$

b) Find the values of $f'(0), f''(0), f'''(0) \text{and} f''''(0)$.

c) Express $\ln\cos x$ as a series in ascending powers of *x* up to and including the term in x^4.

d) Show that, using the first two terms of the series for $\ln\cos x$, with $x = \frac{\pi}{4}$, gives a value for $\ln 2$ of $\frac{\pi^2}{16}\left(1 + \frac{\pi^2}{96}\right)$.

Answers:

(1) $x < -3$ or $-1 < x < 1$

(2) $\frac{9}{8}\pi a^2$

(3) $\left(\dfrac{2ab}{a+b}, \arccos\dfrac{b-a}{b+a}\right); \left(\frac{3}{4}\pi - 2\right)a^2$

(4a) $A = \frac{1}{2}$

(4b) $x = \left(1 + t + \frac{1}{2}t^2\right)\mathrm{e}^{-t}$

(5a) $x = 2\mathrm{e}^{2t} + t$

(5b) $x = e^{3t} + e^{-3t} - \sin t$

(5c) $x = \left(t + \frac{1}{2}t^3\right)\mathrm{e}^{2t}$

(6) 0.618

(9a) $1 + 3x + \dfrac{9x^2}{2} + \dfrac{9x^3}{2} + \dfrac{27}{8}x^4 + \ldots$

(9b) $2x - 2x^2 + \dfrac{8x^3}{3} - 4x^4 + \ldots$

(9c) $x^2 - \dfrac{x^4}{3}$

(10b) $\mathrm{f}(0) = 0, \mathrm{f}'(0) = -1, \mathrm{f}''(0) = 3, \text{and } \mathrm{f}'''(0) = -2$

(11b) $\mathrm{f}'(0) = 0, \mathrm{f}''(0) = -1, \mathrm{f}'''(0) = 0, \text{and } \mathrm{f}''''(0) = -2$

(11c) $\dfrac{-x^2}{2} - \dfrac{x^4}{12}$

BHASVIC

MαTHS

ASSIGNMENT COVER SHEET eta

Name -- Maths Teacher --

Question	Done	Backpack	Ready for test	Notes
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2				
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