α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	σ	τ	υ	φ	χ	ψ	ω

"Mathematics is indeed dangerous in that it absorbs students to such a degree that it dulls their senses to everything else"

P Kraft

Further Maths A2 (M2FP2D1) Assignment ζ (zeta) A

Due w/b 30th October 17

DRILL *Drills are the very basic techniques you need to solve maths problems. You need to practise these until you can do them quickly and accurately. Answers are not provided for drill questions.*

Sketch the following pairs of curves on the same axes without a calculator

a) $r = a(1+\cos\theta)$ and $r = 3a\cos\theta$

b) $r = 2+\cos\theta$ and $r = 5\cos\theta$

c) $r = 4\cos\theta$ and $r = 2\sec\theta$

d) $r = \sin 2\theta$ and $r = \frac{1}{2}$

e) $r = 3a(1-\cos\theta)$ and $r = a(1+\cos\theta)$

f) $r = a(3+2\cos\theta)$ and $r = a(5-2\cos\theta)$

PREPARATION *Every week you will be required to do some preparation for future lessons, to be advised by your teacher.*

1. Show that for small values of x, $e^{2x} - e^{-x} \approx 3x + \frac{3}{2}x^2$.

2. Find the series expansions, up to and including the term in x^4, of

   a) $\ln(1+x-2x^2)$

   b) $\ln(9+6x+x^2)$

   and in each case give the range of values of x for which the expansion is valid.

3. a) Write down the series expansion of $\cos 2x$ in ascending powers of x, up to and including the term in x^8.

   b) Hence, or otherwise, find the first 4 non-zero terms in the power series for $\sin^2 x$.

4. On the same diagram, sketch the curve C_1 with polar equation

$$r = 2\cos 2\theta, \quad -\tfrac{\pi}{4} < \theta \leqslant \tfrac{\pi}{4}$$

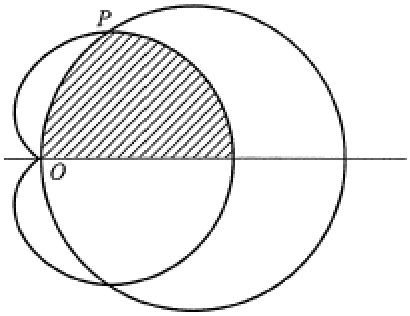
   and the curve C_2 with polar equation $\theta = \frac{\pi}{12}$.
   Find the area of the smaller region bounded by C_1 and C_2.

5. Find using the critical values method the complete set of values of x for which

$$\frac{1+x}{1-x} > \frac{2-x}{2+x}$$

CONSOLIDATION

6.



The diagram shows a sketch of the curves with polar equations

$$r = a(1 + \cos\theta) \quad \text{and} \quad r = 3a\cos\theta, \quad a > 0$$

(a) Find the polar coordinates of the point of intersection P of the two curves.

(b) Find the area, shaded in the figure, bounded by the two curves and by the initial line $\theta = 0$, giving your answer in terms of π. [E]

7. Show that for $x > 1$,

$$\ln(x^2 - x + 1) + \ln(x+1) - 3\ln x = \frac{1}{x^3} - \frac{1}{2x^6} + \ldots + \frac{(-1)^{n-1}}{nx^{3n}} + \ldots$$

8. Sketch the curve C with polar equation

$$r^2 = a^2 \cos 2\theta, \quad 0 \leqslant \theta \leqslant \tfrac{\pi}{4}$$

Find the polar coordinates of the point where the tangent to C is parallel to the initial line $\theta = 0$. [E]

9. Find the set of values of *x* for which

$$\frac{2}{x+2} > \frac{1}{x-3}$$

10. (a) By using the power series expansion for cos *x* and the power series expansion for $\ln(1+x)$, find the series expansion for $\ln(\cos x)$ in ascending powers of *x* up to and including the term in x^4.

(b) Hence, or otherwise, obtain the first two non-zero terms in the series expansion for $\ln(\sec x)$ in ascending powers of *x*.

11. (a) Find the Taylor expansion of cos 2*x* in ascending powers of $\left(x - \frac{\pi}{4}\right)$ up to and including the term in $\left(x - \frac{\pi}{4}\right)^5$.

(b) Use your answer to part **a** to obtain an estimate of cos 2, giving your answer to 6 decimal places.

Answers:

(2a) $x-\frac{5x^2}{2}+\frac{7x^3}{3}-\frac{17x^4}{4}+\ldots,\ \frac{1}{2}<x\le\frac{1}{2}$(smaller interval)

(2b) $2\ln 3+\frac{2x}{3}-\frac{x^2}{9}+\frac{2x^3}{81}-\frac{x^4}{162}+\ldots,\ -3<x\le 3$

(3a) $1-2x^2+\frac{2x^4}{3}-\frac{4x^6}{45}+\frac{2}{315}x^8-\ldots$ (3b) $x^2-\frac{x^4}{3}+\frac{2x^6}{45}-\frac{1}{315}x^8+\ldots$

(4) $\frac{\pi}{6}-\frac{\sqrt{3}}{8}$ (5) $x<-2,\ 0<x<1$ (6) (a) $\left(\frac{3a}{2},\frac{\pi}{3}\right)$ (b) $\frac{5}{8}\pi a^2$

(8) $\left(\frac{a}{\sqrt{2}},\frac{\pi}{6}\right)$ (9) $-2<x<3$ or $x>8$ (10a) $-\frac{x^2}{2}-\frac{x^4}{12}-\ldots$

(10b) $\frac{x^2}{2}+\frac{x^4}{12}+\ldots$ (11a) $-2\left(x-\frac{\pi}{4}\right)+\frac{4}{3}\left(x-\frac{\pi}{4}\right)^3-\frac{4}{15}\left(x-\frac{\pi}{4}\right)^5+\ldots$ (11b) –0.416147 (6 d.p.)

Over the half term break do this Practice Paper completely <u>then</u> check the mark scheme

FP2 PRACTICE PAPER 2

1. Using algebra, find the set of values of x for which

$$2x-5>\frac{3}{x}.$$

(7)

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2. (*a*) Find the general solution of the differential equation

$$2\frac{d^2y}{dt^2}+7\frac{dy}{dt}+3y=3t^2+11t.$$

(8)

(*b*) Find the particular solution of this differential equation for which $y = 1$ and $\frac{dy}{dt} = 1$ when $t = 0$.

(5)

(*c*) For this particular solution, calculate the value of y when $t = 1$.

(1)

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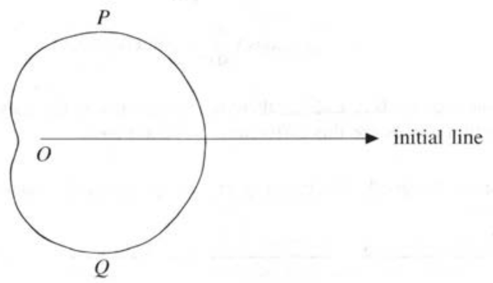
3. **Figure 1**



The curve C shown in Fig. 1 has polar equation

$$r = a(3 + \sqrt{5} \cos \theta), \quad -\pi \leq \theta < \pi.$$

(a) Find the polar coordinates of the points P and Q where the tangents to C are parallel to the initial line.

This means, work out the values of θ for which there is a turning point, ie points at which the change of the y coordinate, as theta changes a tiny bit, is zero.

$$ie)\ \frac{dy}{d\theta} = 0$$

$$ie)\ \frac{d}{d\theta}(y) = 0$$

$$ie)\ \frac{d}{d\theta}(r \sin\theta) = 0$$

(6)

The curve C represents the perimeter of the surface of a swimming pool. The direct distance from P to Q is 20 m.

(b) Calculate the value of a.

(3)

(c) Find the area of the surface of the pool.

This means work out $\frac{1}{2}\int r^2\, d\theta$. The limits are 0 all the way to 2π in order to get all the area. Or you could do 0 to π and double it, since the pool is symmetric.

(6)

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4. $$y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+y=0.$$

(*a*) Find an expression for $\frac{d^3y}{dx^3}$.

(5)

Given that $y = 1$ and $\frac{dy}{dx} = 1$ at $x = 0$,

(*b*) find the series solution for *y*, in ascending powers of *x*, up to an including the term in x^3.

(5)

(*c*) Comment on whether it would be sensible to use your series solution to give estimates for *y* at $x = 0.2$ and at $x = 50$.

(2)

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5. (*a*) Sketch, on the same axes, the graphs with equation $y = |2x - 3|$, and the line with equation $y = 5x - 1$.

(2)

(*b*) Solve the inequality $|2x - 3| < 5x - 1$.

(3)

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6. (*a*) Use the substitution $y = vx$ to transform the equation

$$\frac{dy}{dx} = \frac{(4x + y)(x + y)}{x^2},\ x > 0 \qquad \text{(I)}$$

into the equation

$$x\frac{dv}{dx} = (2 + v)^2. \qquad \text{(II)}$$

(4)

(*b*) Solve the differential equation II to find v as a function of x.

(5)

(*c*) Hence show that

$$y = -2x - \frac{x}{\ln x + c},$$ where c is an arbitrary constant,

is a general solution of the differential equation I.

(1)

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7. (*a*) Find the value of λ for which $\lambda x \cos 3x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 9y = -12 \sin 3x.$$

(4)

(*b*) Hence find the general solution of this differential equation.

(4)

The particular solution of the differential equation for which $y = 1$ and $\frac{dy}{dx} = 2$ at $x = 0$, is $y = g(x)$.

(*c*) Find $g(x)$.

(4)

TOTAL MARKS: 75

Answers:

(2a) $x-\frac{5x^2}{2}+\frac{7x^3}{3}-\frac{17x^4}{4}+\ldots,\ \tfrac{1}{2}<x\le\tfrac{1}{2}$ (smaller interval)

(2b) $2\ln 3+\frac{2x}{3}-\frac{x^2}{9}+\frac{2x^3}{81}-\frac{x^4}{162}+\ldots,\ -3<x\le 3$

(3a) $1-2x^2+\frac{2x^4}{3}-\frac{4x^6}{45}+\frac{2}{315}x^8-\ldots$ (3b) $x^2-\frac{x^4}{3}+\frac{2x^6}{45}-\frac{1}{315}x^8+\ldots$

(4) $\frac{\pi}{6}-\frac{\sqrt{3}}{8}$ (5) $x<-2,\ 0<x<1$ (6) (a) $\left(\frac{3a}{2},\ \frac{\pi}{3}\right)$ (b) $\frac{5}{8}\pi a^2$

(8) $\left(\frac{a}{\sqrt{2}},\ \frac{\pi}{6}\right)$ (9) $-2<x<3$ or $x>8$ (10a) $-\frac{x^2}{2}-\frac{x^4}{12}-\ldots$

(10b) $\frac{x^2}{2}+\frac{x^4}{12}+\ldots$ (11a) $-2\left(x-\frac{\pi}{4}\right)+\frac{4}{3}\left(x-\frac{\pi}{4}\right)^3-\frac{4}{15}\left(x-\frac{\pi}{4}\right)^5+\ldots$ (11b) –0.416147 (6 d.p.)

FP2 PRACTICE PAPER 2 Mark Schemes

1.	$(x>0)$ $2x^2-5x>3$ or $2x^2-5x=3$	M1
	$(2x+1)(x-3)$, critical values $-½$ and 3	A1, A1
	$x>3$	A1 ft
	$x<0$ $2x^2-5x<3$	M1
	Using critical value 0: $-½<x<0$	M1, A1 ft
Alt.	$2x-5-\frac{3}{x}<0$ or $(2x-5)x^2>3x$	M1
	$\frac{(2x+1)(x-3)}{x}>0$ or $x(2x+1)(x-3)>0$	M1, A1
	Critical values $-½$ and 3, $x>3$	A1, A1 ft
	Using critical value 0, $-½<x<0$	M1, A1 ft
		(7 marks)

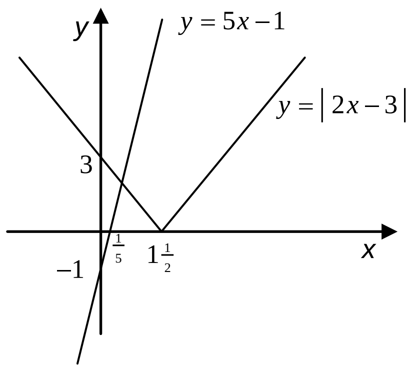
2. (*a*)	$2m^2+7m+3=0$ $(2m+1)(m+3)=0$	
	$m=-½,\ -3$	
	C.F. is $y=Ae^{-½t}+Be^{-3t}$	M1, A1
	P.I. $y=at^2+bt+c$	B1
	$y'=2at+b$, $y''=2a$	

	$2(2a) + 7(2at + b) + 3(at^2 + bt + c) \equiv 3t^2 + 11t$	M1	
	$3a = 3,\quad a = 1 \qquad 14 + 3b = 11\ ,\quad b = -1$	A1	
	$4 - 7 + 3c = 0,\quad c = 1$	M1, A1	
	General solution: $y = Ae^{-\frac{1}{2}t} + Be^{-3t} + (t^2 - t + 1)$	A1 ft	**(8)**
(*b*)	$y' = -\frac{1}{2}Ae^{-\frac{1}{2}t} - 3Be^{-3t} + (2t - 1)$	M1	
	$t = 0,\ y' = 1:\quad 1 = -1 - \frac{1}{2}A - 3B$		
	$t = 0, y = 1:\quad 1 = 1 + A + B$ one of these	M1, A1	
	Solve: $A + B = 0,\quad A + 6B = -4$		
	$A = {}^4/_5,\ B = -{}^4/_5$	M1	
	$y = (t^2 - t + 1) + \frac{4}{5}(e^{-\frac{1}{2}t} - e^{-3t})$	A1	**(5)**
(*c*)	$t = 1:\quad y = \frac{4}{5}(e^{-\frac{1}{2}} - e^{-3}) + 1 \qquad (= 1.445\ldots)$	B1	**(1)**
			(14 marks)

3.	(*a*)	$y = r\sin\theta = a(3\sin\theta + \sqrt{5}\sin\theta\cos\theta)$		
		$\frac{dy}{d\theta} = a(3\cos\theta + \sqrt{5}\cos 2\theta)$	M1, A1	
		$2\sqrt{5}\cos^2\theta + 3\cos\theta - \sqrt{5} = 0$		
		$\cos\theta = \frac{-3 \pm \sqrt{9 + 40}}{4\sqrt{5}}\ ,\quad \cos\theta = \frac{1}{\sqrt{5}}$	M1, A1	
		$\theta = \pm 1.107\ldots$	A1 ft	
		$r = 4a$	A1 ft	**(6)**
	(*b*)	$2r\sin\theta = 20$	M1	
		$8a\sin\theta = 20,\quad a = \frac{20}{8\sin\theta} = 2.795\ldots$	M1, A1	**(3)**
	(*c*)	$(3 + \sqrt{5}\cos\theta)^2 = 9 + 6\sqrt{5}\cos\theta + 5\cos^2\theta$	B1	
		Integrate: $9\theta + 6\sqrt{5}\sin\theta + 5\left(\frac{\sin 2\theta}{4} + \frac{\theta}{2}\right)$	M1, A1	
		Limits used: $\left[\ldots\right]_0^{2\pi} = 18\pi + 5\pi$ (or upper limit: $9\pi + \frac{5\pi}{2}$)	A1	

	$\frac{1}{2}\int_0^{2\pi} r^2 d\theta = a^2(23\pi) \quad \approx 282 \text{ m}^2$	M1, A1 **(6)** **(15 marks)**

4. (*a*)	$y\frac{d^3y}{dx^3} + \frac{dy}{dx}\frac{d^2y}{dx^2}; + 2\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}; + \frac{dy}{dx} = 0$ marks can be awarded in(b)	M1 A1; B1;B1
	$\frac{d^3y}{dx^3} = \frac{-3\frac{dy}{dx}\frac{d^2y}{dx^2} - \frac{dy}{dx}}{y}$ or sensible correct alternative	B1 **(5)**
(*b*)	When $x = 0$ $\frac{d^2y}{dx^2} = -2$, and $\frac{d^3y}{dx^3} = 5$	M1A1, A1 ft
	$\therefore \quad y = 1 + x - x^2 + \frac{5}{6}x^3 ...$	M1, A1 ft **(5)**
(*c*)	Could use for $x = 0.2$ but not for $x = 50$ as	B1
	approximation is best at values close to $x = 0$	B1 **(2)**
		(12 marks)

5. (*a*)	 shape points on axes	B1 B1 **(2)**
(*b*)	$-2x + 3 = 5x - 1$	M1
	$x = \frac{4}{7}$	A1
	$x > \frac{4}{7}$	A1 ft **(3)**
		(5 marks)

6. (*a*)	$v + x\frac{dv}{dx}$,= $(4 + v)(1 + v)$	M1, M1

	$x\frac{\mathrm{d}v}{\mathrm{d}x} = v^2 + 5v + 4 - v$	A1	
	$x\frac{\mathrm{d}v}{\mathrm{d}x} = (v + 2)^2$ *	A1	**(4)**
(*b*)	$\int \frac{1}{(v+2)^2}\,\mathrm{d}v = \int \frac{1}{x}\,\mathrm{d}x$	B1, M1	
	$-\frac{1}{2+v} = \ln x + c$ must have + *c*	M1 A1	
	$2 + v = -\frac{1}{\ln x + c}$	M1	
	$v = -\frac{1}{\ln x + c} - 2$	A1	**(5)**
(*c*)	$y = -2x - \frac{x}{\ln x + c}$	B1	**(1)**
			(10 marks)

7.	(*a*)	$y = \lambda x \cos 3x$	
		$\frac{dy}{dx} = \lambda \cos 3x - 3\lambda x \sin 3x$	M1 A1
		$\frac{d^2y}{dx^2} = -3\lambda \sin 3x - 3\lambda \sin 3x - 9\lambda x \cos 3x$	A1
		$\therefore\ -6\lambda \sin 3x - 9\lambda x \cos 3x + 9\lambda x \cos 3x = -12 \sin 3x$	
		$\lambda = 2$ cso	A1 **(4)**
	(*b*)	$\lambda^2 - 9 = 0$	M1
		$\lambda = (\pm)3\text{i}$	A1
		$\therefore\ y = A \sin 3x + B \cos 3x$ form	M1
		$\therefore\ y = A \sin 3x + B \cos 3x + 2x \cos 3x$	A1 ft on λ's **(4)**
	(*c*)	$y = 1, x = 0 \Rightarrow B = 1$	B1
		$\frac{dy}{dx} = 3A \cos 3x - 3B \sin 3x + 2 \cos 3x - 6x \sin 3x$	M1 A1ft on λ's
		$2 = 3A + 2 \Rightarrow A = 0$	
		$\therefore\ y = \cos 3x + 2x \cos 3x$	A1 **(4)**
			(12 marks)

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MαTHS

ASSIGNMENT COVER SHEET zeta

Name ________________________ Maths Teacher ________________________

Question	Done	Backpack	Ready for test	Notes
Drill				
1				
2				
3				
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