α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	σ	τ	υ	φ	χ	ψ	ω

"It is a mathematical fact that the casting of this pebble from my hand alters the centre of gravity of the universe."

T Carlyle

Further Maths A2 (M2FP2D1) Assignment ε (epsilon) A

Due w/b 16th Oct

DRILL *Drills are the very basic techniques you need to solve maths problems. You need to practise these until you can do them quickly and accurately. Answers are not provided for drill questions.*

Sketch the following polar curves and lines

a) $r = a(1+\theta)$ b) $r = a(1-\cos\theta)$ c) $r = a(3+2\cos\theta)$

d) $r^2\cos 2\theta = a^2$ e) $r = 2+\cos\theta$ f) $r = a$

g) $r = 2\sin 3\theta$ h) $p = r\cos(\alpha-\theta)$ i) $\theta = \frac{\pi}{3}$

j) $r = ae^{\theta}$ k) $r = 2a\sec\theta$ l) $r = 4\cos 2\theta$

PREPARATION *Every week you will be required to do some preparation for future lessons, to be advised by your teacher.*

CURRENT WORK AND CONSOLIDATION

FP2: Solving Inequalities

1 Find the set of values of x for which:

(a) $2x-1 < 4(x-3)$ (b) $(x-1)(x-5) > 0$ (c) $x^2 < 3x+4$

FP2: Polar Co-ordinates

2 Transform to polar form the circles with equations

$x^2+y^2+2x=0$ and $x^2+y^2-2y=0$

3 Obtain the Cartesian form of the polar equations where a is a positive constant

a) $r = a\tan\theta\sec\theta$ b) $r = 2a\tan\theta$ c) $r = a(1-\cos\theta)$

FP2 Polar Coordinates & Polar Curves

4 Given that the origin *O* and the pole coincide and that the initial line is *Ox*, find the polar coordinates of the point whose Cartesian coordinates are:

(a) (3, 0) (b) (0, 4) (c) (–3.5, 0)

(d) (0, –5) (e) (3, 4) (f) (–4, 3)

5 Given that the pole *O* and the origin coincide and that the positive *x*-axis is the initial line, find the Cartesian coordinates of the points whose polar coordinates are:

(a) (3, 0) (b) $\left(2, \frac{\pi}{2}\right)$ (c) $\left(5, -\frac{\pi}{2}\right)$

(d) $\left(2, \pi\right)$ (e) $\left(3, \frac{\pi}{3}\right)$ (f) $\left(4, \frac{2\pi}{3}\right)$

6 Sketch the lines (or half-lines) with polar equations:

(a) $\theta = -\frac{\pi}{3}$ (b) $\theta = 3$ (c) $r\cos\theta = 2$

(d) $r\sin\theta = -2$ (e) $r\cos\left(\theta - \frac{\pi}{6}\right) = 2$ (f) $r\cos\left(\theta + \frac{\pi}{6}\right) = 4$

FP2 Inequalities

7 Prove that for all real *x*

$$\left|\frac{x+1}{x^2+2x+2}\right| \le \tfrac{1}{2}$$

8 Find the set of values of *x* for which

$$\frac{2}{x+2} > \frac{1}{x-3}$$

9 (a) Express $\frac{4}{u^2-4}$ in partial fractions.

The function *y(x)* satisfies the differential equation

$$4x\left(\frac{dy}{dx}+1\right) - y = \frac{y^2-3x^2}{x} \qquad (1) \qquad \text{where } x > 0$$

(b) Show that the substitution $y = ux$ transforms equation (1) into the differential equation

$$4\frac{du}{dx} = \frac{u^2-4}{x} \qquad (2)$$

(c) Show that the general solution of equation (2) is given by

$$u = 2\left(\frac{kx+1}{1-kx}\right) \quad \text{for } k \text{ an arbitrary constant.}$$

(d) Find the particular solution of equation (1) given that $y = -3$ at $x = 3$.

10 (a) Show that the substitution $y = ux$ transforms the differential equation

$$(y-2x)\frac{\mathrm{d}y}{\mathrm{d}x} = 2y - x \qquad (1)$$

into the differential equation

$$\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{u^2-4u+1}{x(2-u)} \qquad (2)$$

(b) Find the general solution of equation (2).

(c) Hence show that the general solution of equation (1) satisfies the equation

$$y = 2x \pm \sqrt{A+3x^2} \quad \text{for } A \text{ an arbitrary constant.}$$

(d) A particular solution of equation (1) is such that $y = 4$ when $x = 1$.

(i) Find the equation of this particular solution in the form $y = \mathrm{f}(x)$

(ii) Deduce that large values of x, $\mathrm{f}(x)$ can be approximated by the expression $\left(2+\sqrt{3}\right)x$

11 (a) Find $\int \frac{1}{x\ln x}\,\mathrm{d}x$

(b) Use the substitution $y = ux$ to show that the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} = y(\ln y - \ln x + 1) \quad \text{where } x, y \geq 0 \qquad (1)$$

can be transformed into the equation

$$\frac{du}{dx} = \frac{u\ln u}{x} \qquad (2)$$

(c) Find the general solution of equation (2).

(d) (i) Find the particular solution of equation (1) for which

$$y = \frac{1}{\mathrm{e}^2} \quad \text{when } x = 1$$

(ii) Sketch the graph of this particular solution for $x \geq 0$.
You may assume this curve has exactly one stationary point.

12 (a) Find the general solution of the differential equation

$$3\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + 2\frac{\mathrm{d}y}{\mathrm{d}t} - y = (t+3)(t-4)$$

(b) Find the particular solution of this differential equation for which $y = 5$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = -4$ when $t = 0$.

(c) For this particular solution, calculate the exact value of y when $t = -3$.

13 Given that $2x\sin 3x$ is a particular integral of the differential equation

$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 9y = k\cos 3x$ where k is a constant.

(a) Show that $k = 12$

(b) Find the particular solution of this differential equation for which $y = 0$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 6$ at $x = 0$.

The equation of a curve C is given by this particular solution.

(c) Find the exact coordinates of the points where C crosses the x-axis in the interval $-\frac{1}{3}\pi \leq x \leq \frac{1}{3}\pi$.

14 (a) Verify that $y = 2x\mathrm{e}^{-x}$ is a particular integral of the differential equation

$$2\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 3\frac{\mathrm{d}y}{\mathrm{d}x} + y = -2\mathrm{e}^{-x}$$

(b) Find the particular solution of this differential equation for which $y = -1$ when $x = 0$ and for which $y = \ln 2$ when $x = \ln 4$.

(c) Hence find the exact value of y when $x = 1$.

15 (a) Show that the transformation $y = xv$ transforms the equation

$$x^2\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 2x\frac{\mathrm{d}y}{\mathrm{d}x} + 2\left(1+8x^2\right)y = 8x^5 + x^3 \qquad (1)$$

into the equation

$$\frac{\mathrm{d}^2 v}{\mathrm{d}x^2} + 16v = 8x^2 + 1 \qquad (2)$$

(b) Solve equation (2) to find v as a function of x.

(c) Hence state the general solution of equation (1).

Answers:

(1a) $x > 5\frac{1}{2}$ (1b) $x < 1$ or $x > 5$ (1c) $-1 < x < 4$

(2) $r + 2\cos\theta = 0, r = 2\sin\theta$

(3a) $x^2 = ay$ (3b) $x^2\left(x^2 + y^2\right) = 4a^2y^2$ (3c) $\left(x^2 + y^2 + ax\right)^2 = a^2\left(x^2 + y^2\right)$

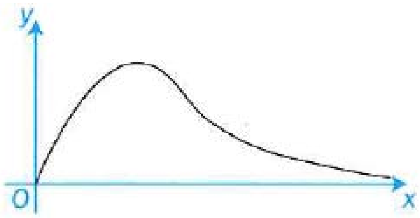
(4a) (3, 0) (4b) $\left(4, \frac{\pi}{2}\right)$ (4c) (3.5, π) (4d) $\left(5, -\frac{\pi}{2}\right)$ (4e) (5, 0.927) (4f) (5, 2.498)

(5a) (3, 0) (5b) (0, 2) (5c) (0, –5) (5d) (–2, 0) (5e) $\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$ (5f) (–2, 2√3)

(8) $-2 < x < 3$, or $x > 8$ (9a) $\frac{1}{(u-2)} - \frac{1}{(u+2)}$ (9d) $y = \frac{2x(1-x)}{(1+x)}$

(10b) $u = 2 \pm \frac{1}{x}\sqrt{3x^2 + A}$ (10d i) $y = 2x \pm \sqrt{3x^2 + 1}$ (11a) $\int \frac{1}{x\ln x}\,dx = \ln\left|\ln|x|\right| + c$

(11c) $u = \mathrm{e}^{Ax}$, for $A \in \mathbb{R}$ (11d i) $y = x\mathrm{e}^{-2x}$ (11d ii)



(12a) $y = A\mathrm{e}^{\frac{1}{3}t} + B\mathrm{e}^{-t} - t^2 - 3t$ (12b)

$y = 3\mathrm{e}^{\frac{1}{3}t} + 2\mathrm{e}^{-t} - t^2 - 3t$ (12c) $y = 2\mathrm{e}^3 + 3\mathrm{e}^{-1}$

(13b) $y = 2(1+x)\sin 3x$ (13c) $\left(\pm\frac{1}{3}\pi, 0\right), (-1, 0)$

(14b) $y = \mathrm{e}^{-\frac{1}{2}x} - 2\mathrm{e}^{-x} + 2x\mathrm{e}^{-x}$ (14c) $y = -\mathrm{e}^{-\frac{1}{2}}$ (15b) $v = A\cos 4x + B\sin 4x + \frac{1}{2}x^2$

(15c) $y = x\left(A\cos 4x + B\sin 4x + \frac{1}{2}x^2\right)$

STEP QUESTIONS

Try this STEP question.

Information for candidates on the exam papers is as follows:

- Each question is marked out of 20. There is no restriction of choice.
- You will be assessed on the **six** questions for which you gain the highest marks.
- You are advised to concentrate on no more than **six** questions. Little credit will be given to fragmentary answers.
- You are provided with Mathematical Formulae and Tables.
- **Electronic calculators are not permitted.**

1 (i) Express $\left(3+2\sqrt{5}\right)^3$ in the form $a+b\sqrt{5}$ where a and b are integers.

(ii) Find the positive integers c and d such that $\sqrt[3]{99-70\sqrt{2}}=c-d\sqrt{2}$

(iii) Find the two real solutions of $x^6-198x^3+1=0$

STEP 1: June 2004

BHASVIC

MαTHS

ASSIGNMENT COVER SHEET epsilon

Name -- Maths Teacher --

Question	Done	Backpack	Ready for test	Notes
Drill				
1				
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