α	β	$\boxed{\gamma}$	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	σ	τ	υ	φ	χ	ψ	ω

"All maths is easy"

D McDonnell

Further Maths A2 (M2FP2D1) Assignment γ (gamma) A

Come along to the Maths Association Talk – 29th September at 4:30pm in room 3
"Geometry Ancient And Modern" (It's free! Come along!)

Maths Trip: Maths In Action University Lectures in London. £20 a ticket (10 tickets available) 15th November
Maths Trip: Maths In Action University Lectures in London. £20 a ticket (10 tickets available) 14th December

DRILL

Drills are the very basic techniques you need to solve maths problems. You need to practise these until you can do them quickly and accurately. Answers are not provided for drill questions.

A Find the Cartesian equations for each of the following

1.
$$x = t + 1$$
$$y = 4 - 3t$$

2.
$$x = 2t + 1$$
$$y = 3t - 2$$

3.
$$x = \frac{2}{t}$$
$$y = 2t - 1$$

4.
$$x = 2t + 1$$
$$y = t^2 - 1$$

B Find the Cartesian equation of each of the following

1.
$$x = 5\cos t$$
$$y = 5\sin t$$

2.
$$x = 1 + \cos\theta$$
$$y = 2 + \sin\theta$$

3.
$$x = 2\tan\theta$$
$$y = \cos\theta$$

4.
$$x = \cos t$$
$$y = \operatorname{cosec} t$$

PREPARATION

Every week you will be required to do some preparation for future lessons, to be advised by your teacher.

CURRENT WORK AND CONSOLIDATION

2 inequalites

1. Use the substitution $u = x + y$ to find a general solution of the differential equation
$$\frac{dy}{dx} = x + y$$

2. Use the substitution $v = x + 2y$, to find the particular solution of the differential equation
$$\frac{dy}{dx} = x + 2y$$
Given that $y = -\frac{1}{4}$ at $x = 0$

3. Use the substitution $v = \frac{y}{x}$, to solve the differential equation

$$\frac{dy}{dx}=\frac{y}{x}-\left(\frac{y}{x}\right)^2, x>0$$

Subject to the condition $y=1$ at $x=1$

4. Given

$$\frac{dy}{dx}=\frac{(4x+y)(x+y)}{x^2}, x>0$$

Use the substitution $y=xv$, to show that the above differential equation can be transformed to

$$x\frac{dv}{dx}=(v+2)^2$$

Hence find the particular solution to the differential equation give n $y=-1$ at $x=1$

5. Use the substitution $y=\frac{z}{x}$ to transform the differential equation

$x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+(2-4x)\frac{\mathrm{d}y}{\mathrm{d}x}-4y=0$ into the equation $\frac{\mathrm{d}^2z}{\mathrm{d}x^2}-4\frac{\mathrm{d}z}{\mathrm{d}x}=0$.

Hence solve the equation $x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+(2-4x)\frac{\mathrm{d}y}{\mathrm{d}x}-4y=0$, giving y in terms of x.

6. Find the general solution of each differential equation using the substitution $x=\mathrm{e}^u$, where u is a function of x.

a) $x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+6x\frac{\mathrm{d}y}{\mathrm{d}x}+4y=0$

b) $x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+5x\frac{\mathrm{d}y}{\mathrm{d}x}+4y=0$

c) $x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+6x\frac{\mathrm{d}y}{\mathrm{d}x}+6y=0$

d) $x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+4x\frac{\mathrm{d}y}{\mathrm{d}x}-28y=0$

e) $x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-4x\frac{\mathrm{d}y}{\mathrm{d}x}-14y=0$

f) $x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+3x\frac{\mathrm{d}y}{\mathrm{d}x}+2y=0$

7. Use the substitution $y=\frac{z}{x^2}$ to transform the differential equation

$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2x(x+2)\frac{\mathrm{d}y}{\mathrm{d}x}+2(x+1)^2y=\mathrm{e}^{-x}$ into the equation $\frac{\mathrm{d}^2z}{\mathrm{d}x^2}+2\frac{\mathrm{d}z}{\mathrm{d}x}+2z=\mathrm{e}^{-x}$.

Hence solve the equation $x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2x(x+2)\frac{\mathrm{d}y}{\mathrm{d}x}+2(x+1)^2y=\mathrm{e}^{-x}$, giving y in terms of

8. Use the substitution $z=\sin x$ to transform the differential equation

$\cos x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\sin x\frac{\mathrm{d}y}{\mathrm{d}x}-2y\cos^3x=2\cos^5x$ into the equation $\frac{\mathrm{d}^2y}{\mathrm{d}z^2}-2y=2(1-z^2)$.

Hence solve the equation $\cos x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\sin x\frac{\mathrm{d}y}{\mathrm{d}x}-2y\cos^3x=2\cos^5x$, giving y in terms of x.

9. The curve C is given by the parametric equations

$$x = \frac{2}{t}, \qquad y = 4t, \qquad t > 0$$

The tangent to C at point P where $t = p$, meets the coordinate axes at the points A and B.

Show that the area of the triangle OAB, where O is the origin is independent of p.

10. The curve C is given parametrically by the equations

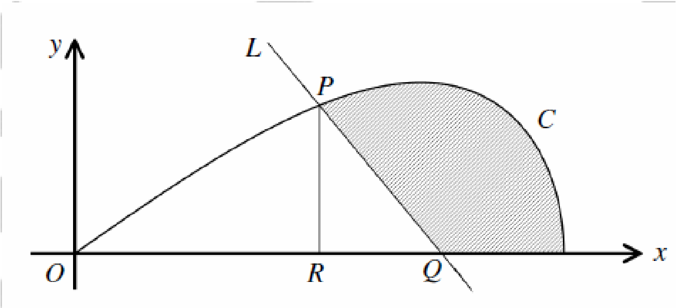
$$x = 2t, \qquad y = 8t^3 + 4t^2$$

a) Find the coordinates of the stationary points of C, and determine their nature.

It is further given that C has a single point of inflection at P

b) Determine the coordinates of P

11. The figure below shoes the curve C with parametric equations $x = 6\cos t\,, y = 3\sin 2t\,, 0 \le t \le \frac{\pi}{2}$



The point P lies on C where $x = 3$

a) Find the y coordinate of P

The line L is the normal to the curve at P. This normal meets the x axis at Q.

b) Show that an equation of L is

$$2y + 2x\sqrt{3} = 9\sqrt{3}$$

The line PR is parallel to the y axis

c) Show that the area of the finite region bounded by C, the line PR and the x axis is given by the integral

$$\int_0^{\frac{\pi}{2}} 36\sin^2 t\,dt$$

d) Hence find an exact value of the shaded region, bounded by C, the normal L and the x axis

12.

a) Simplify $f(r)$ into a single fraction

$$f(r) = \frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}$$

b) Hence show that

$$\sum_{r=1}^{20} \frac{1}{r(r+1)(r+2)} = \frac{115}{462}$$

13. G

$$f(r) = \frac{1}{r(r+2)}$$

a) Express $f(r)$ in partial fractions

b) Hence prove, by the method of difference, that

$$\sum_{1}^{n} f(r) = \frac{n(An+B)}{4(n+1)(n+2)}$$

Where A and B are constants to be found

14. Find the set of values of x that satisfy the inequality

$$\left|\frac{(x-1)(x+4)}{x^2+4}\right| < 1$$

15. Determine the range of values of x that satisfy the inequality

$$\left|\frac{x+3}{x}\right| \geq \left|\frac{x}{2-x}\right|$$

Answers:

1. $y = Ae^{x} - x - 1$
2. $y = -\frac{1}{4}(2x+1)$
3. $y = \frac{x}{1+\ln x}$
4. $y = \frac{x}{1-\ln x} - 2x$
5. $y = \frac{A}{x} + \frac{B}{x}e^{4x}$
6.

a) $y = \frac{A}{x^4} + \frac{B}{x}$

b) $y = (A + B\ln x) \times \frac{1}{x^2}$

c) $y = \frac{A}{x^2} + \frac{B}{x^3}$

d) $y = \frac{A}{x^7} + Bx^4$

e) $y = Ax^7 + \frac{B}{x^2}$

f) $y = \frac{1}{x}[A\cos\ln x + B\sin\ln x]$

7. $y = \frac{e^{-x}}{x^2}(A\cos x + B\sin x + 1)$
8. $y = Ae^{\sqrt{2}\sin x} + Be^{-\sqrt{2}\sin x} + \sin^2 x$
9. Proof
10. Min (1,0)
    Max $\left(\frac{1}{3}, \frac{4}{27}\right)$
    $P\left(\frac{1}{3}, \frac{2}{27}\right)$
11. $\frac{3\sqrt{3}}{2}$
    Area=$\frac{27\sqrt{3}}{8}$
12. Proof

13. $A = 3, B = 5$

14. $x < -\frac{3}{2}$ or $0 < x < \frac{8}{3}$

15. $\boldsymbol{-2 \leq x \leq 0}$ **and** $\boldsymbol{0 < x \leq \frac{3}{2}}$ **and** $\boldsymbol{x \geq 6}$

BHASVIC

Maths

ASSIGNMENT ______________________ COVER SHEET

Name -- Maths Teacher --

Question	Done	Backpack	Ready for test	Notes
Drill A				
Drill B				
1				
2				
3				
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