α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	σ	τ	υ	φ	χ	ψ	ω

"The mathematician's patterns, like the painter's or the poet's, must be beautiful: the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test." G H Hardy

Further Maths A2 (M2FP2D1) Assignment ω (omega) A VJM adapted

PREPARATION *Every week you will be required to do some preparation for future lessons, to be advised by your teacher.*
Corrections to the FP2 mock exam should now have been completed and the action questions done

CURRENT WORK – MECHANICS

You need to complete the following past paper, timed and using good exam technique. Check the mark scheme only after you have completed the paper: **M2 Edexcel January 2007.**

DECISION MATHS

D1 June 16 questions 1,2,3,5,6,7 give them in marked with this assignment plus the two route inspection questions in this assignment

CONSOLIDATION – FP2

1. A scientist is modelling the amount of a chemical in the human bloodstream. The amount x o the chemical, measure in mg l^{-1}, at time t hours satisfies the differential equation

$$2x\frac{\mathrm{d}^2x}{\mathrm{d}t^2}-6\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2=x^2-3x^4,\ x>0$$

a) Show that the substitution $y=\frac{1}{x^2}$ transforms this differential equation into $\frac{\mathrm{d}^2y}{\mathrm{d}t^2}+y=3$ [I].

b) Find the general solution of differential equation [I].

Given that at time $t=0,\ x=\frac{1}{2}$ and $\frac{\mathrm{d}x}{\mathrm{d}t}=0$,

c) find an expression for x in term of t,

d) write down the maximum value of x and t varies.

2. Find the set of values of x for which $\frac{x+1}{2x-3}<\frac{1}{x-3}$

3. $\frac{\mathrm{d}y}{\mathrm{d}x}-y\tan x=2\sec^3 x$

Given that y = 3 at x = 0, find y in terms of x.

4. a) Sketch the curve C with polar equation $r = 5 + \sqrt{3}\cos\theta,\quad 0 \le \theta < 2\pi$.

b) Find the polar coordinates of the points where the tangents to C are parallel to the initial line $\theta = 0$. Give your answers to 3 significant figures where appropriate.

c) Using integration, find the area enclosed by the curve c, giving your answer in terms of π .

5. The transformation T from the z-plane, where $z = x + \mathrm{i}\,y$, to the w-plane, where $w = u + \mathrm{i}\,v$, is given by $w = \dfrac{z+\mathrm{i}}{z},\quad z \ne 0$.

a) The transformation T maps the points on the line with equation $y = x$ in the z-plane, other than (0, 0), to points on a line l in the w-plane. Find a Cartesian equation of l.

b) Show that the image, under T, of the line with equation $x + y + 1 = \tilde{0}$ in the z-plane is a circle C in the w-plane, where C has Cartesian equation $u^2 + v^2 - u + v = 0$.

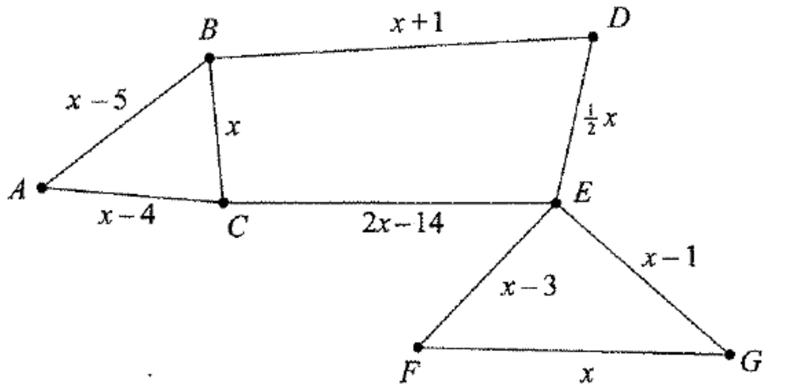
c) On the same Argand diagram, sketch l and C.

6.

(a) Explain why it is impossible to draw a network with exactly three odd vertices.

(2)

Figure 1



The Route Inspection problem is solved for the network in Fig. 1 and the length of the route is found to be 100.

(b) Determine the value of x, showing your working clearly.

7.

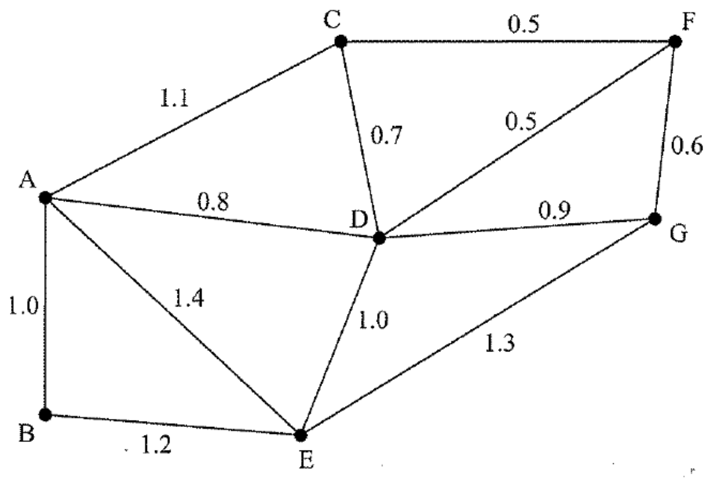


Figure 4

Figure 4 models a network of roads in a housing estate. The number on each arc represents the length, in km, of the road.

The total weight of the network is 11 km.

A council worker needs to travel along each road once to inspect the road surface. He will start and finish at A and wishes to minimise the length of his route.

(a) Use an appropriate algorithm to find a route for the council worker. You should make your method and working clear. State your route and its length.

(6)

A postal worker needs to walk along each road twice, once on each side of the road. She must start and finish at A. The length of her route is to be minimised. You should ignore the width of the road.

(b) (i) Explain how this differs from the standard route inspection problem.

(1)

(ii) Find the length of the shortest route for the postal worker.

(2)

CHALLENGE QUESTION

Sketch, without calculating the stationary points, the graph of the function $\mathrm{f}(x)$ given by

$$\mathrm{f}(x) = (x-p)(x-q)(x-r)\ ,$$

where $p < q < r$. By considering the quadratic equation $\mathrm{f}'(x) = 0$, or otherwise, show that

$$(p+q+r)^2 > 3(qr + rp + pq)\ .$$

By considering $(x^2 + gx + h)(x - k)$, or otherwise, show that $g^2 > 4h$ is a sufficient condition but not a necessary condition for the inequality

$$(g-k)^2 > 3(h - gk)$$

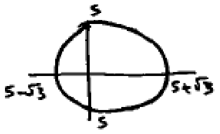
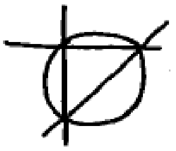
to hold.

Answers:

Current work – M2 January 2007:

1a)	50	1b)	0.32	2a)	8.6kW
2b)	T $\approx$ 21	3a)	$\bar{x} = 25$	3b)	$M = \frac{3}{11}m$
4a)	$u = 8v$	4b)	$k = 3$	4c)	$e = \frac{3}{4}$
4d)	$y = \frac{9}{8}v > v \Rightarrow$ further collision between P and Q	5a)	$\frac{10}{3}mg$	5b)	$\frac{8}{3}mg$
5c)	$\frac{3}{8}$	6a)	$\mathbf{a} = \left(3t^2 - 6\right)i + 4t\mathbf{j}$	6b)	v=9**i**+15**j**
6c)	6.5	6d)	157°	7a)	17.5
7b)	55°	7c)	60		

Consolidation

1b)	$y = A\cos t + B\sin t + 3$	1c)	$x = \frac{1}{\sqrt{3+\cos t}}$	1d)	$\frac{1}{\sqrt{2}} = 0.707$ (3dp)
2)	$0 < x < \frac{3}{2}$, $3 < x < 4$	3)	$y = \frac{2\tan x + 3}{\cos x}$	4a)	
4b)	$\left(\frac{11}{2}, 1.28\right), \left(\frac{11}{2}, 5.01\right)$	4c)	$\frac{5\sqrt{3}}{2}\pi$	5a)	$v = u - 1$
5c)					

6)a) each arc contributes 2 to the sum of degrees, hence this sum must be even. Therefore there must be an even (or zero) number of vertices of odd degree.

b) If x > 9, 10 ½ x -26 = 100, x = 12

(If x < 9, 11 ½ x -35 =110, x = 11 17/23 is inconsistent)

7)(a) CD + FG = 0.7 + 0.6 = 1.3

CF + DG = 0.5 + 0.9 = 1.4

CG + DF = 1.1 +0.5 = 1.6

Repeat CD and FG, give a possible route

Length 11 + 1.3 = 12.3 km

b) i) each arc has to be traversed twice ii) 2 x 11 = 22km

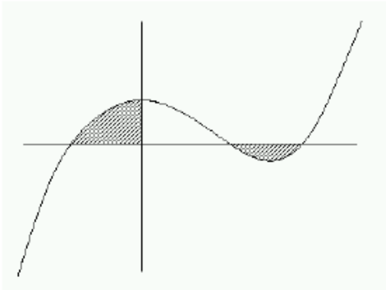
Challenge question: upsilon

(i) We would like to multiply both sides of the inequality by x in order to obtain a nice cubic expression. However, we have to allow for the possibility that x is negative (which would reverse the inequality). One way is to consider the cases $x > 0$ and $x < 0$ separately.

For $x > 0$, we multiply the whole equation by x without changing the inequality:

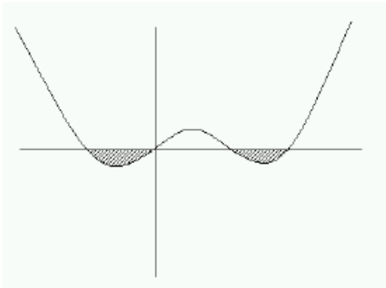
$$\begin{aligned} 1 + 2x - x^2 > 2/x &\Rightarrow x + 2x^2 - x^3 > 2 \\ &\Rightarrow x^3 - 2x^2 - x + 2 < 0 \\ &\Rightarrow (x-1)(x+1)(x-2) < 0 \\ &\Rightarrow 1 < x < 2, \end{aligned} \qquad (*)$$

discarding the possibility $x < -1$, since we have assumed that $x > 0$. The easiest way of obtaining the result $(*)$ from the previous line is to sketch the graph of $(x-1)(x+1)(x-2) < 0$.



For $x < 0$, we must reverse the inequality when we multiply by x so in this case, $(x-1)(x+1)(x-2) > 0$, which gives $-1 < x < 0$.

The smart way to do $x > 0$ and $x < 0$ in one step is to multiply by x^2 (which is never negative and hence never changes the direction of the inequality) and analyse $x(x-1)(x+1)(x-2) < 0$. Again, a sketch is useful.



(ii) First square both sides the inequality $\sqrt{(3x+10)} > 2 + \sqrt{(x+4)}$:

$$3x + 10 > 4 + 4(x+4)^{\frac{1}{2}} + (x+4) \quad \text{i.e.} \quad x + 1 > 2(x+4)^{\frac{1}{2}}.$$

Note that the both sides of the original inequality are positive or zero (i.e. non-negative), so the direction of the inequality is not changed by squaring. Now consider the new inequality $x + 1 > 2(x+4)^{\frac{1}{2}}$. If both sides are non-negative, that is if $x > -1$, we can square both sides again without changing the direction of the inequality. But, if $x < -1$, the inequality cannot be satisfied since the right hand side is always non-negative. Squaring gives

$$x^2 + 2x + 1 > 4(x+4) \quad \text{i.e.} \quad (x-5)(x+3) > 0\,.$$

Thus, $x > 5$ or $x < -3$. However, we must reject $x = -3$ because of the condition $x > -1$. Therefore the inequality holds for $x > 5$.

BHASVIC

MαTHS

ASSIGNMENT COVER SHEET omega

Name ______________________ Maths Teacher ______________________

Question	Done	Backpack	Ready for test	Notes
M2 Edexcel January 2007				
Decision Solomon D				
1				
2				
3				
4				
5				