α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	σ	τ	υ	φ	χ	$\boxed{\psi}$	ω

"The mathematician's patterns, like the painter's or the poet's, must be beautiful: the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test." G H Hardy

Further Maths A2 (M2FP2D1) Assignment ψ (psi) A
Due w/b 19th March 18

PREPARATION *Every week you will be required to do some preparation for future lessons, to be advised by your teacher.*

CURRENT WORK – MECHANICS

You need to complete the following past paper, timed and using good exam technique. Check the mark scheme only after you have completed the paper: **M2 Edexcel Summer 2006.**

CONSOLIDATION – FP2

1. Given that $3x\sin 2x$ is a particular integral of the differential equation $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+4y=k\cos 2x$, where k is a constant,

   a) calculate the value of k,

   b) find the particular solution of the differential equation for which at $x = 0$, $y = 2$, and for which at $x=\frac{\pi}{4}, y=\frac{\pi}{2}$.

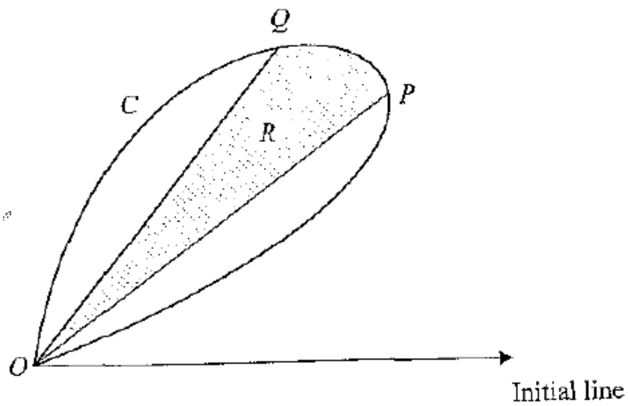
2. a) Use algebra to find the exact solutions of the equation $\left|2x^2+x-6\right|=6-3x$.

   b) On the same diagram, sketch the curve with equation $y=\left|2x^2+x-6\right|$ and the line with equation $y = 6 - 3x$.

   c) Find the set of values of x for which $\left|2x^2+x-6\right|>6-3x$.

3. Obtain the general solution of the differential equation $x\frac{\mathrm{d}y}{\mathrm{d}x}+2y=\cos x,\quad x>0$ giving your answer in the form $y=\mathrm{f}\left(x\right)$.

4.



The figure above shows a sketch of the curve *C* with polar equation

$r = 4\sin\theta\cos^2\theta, \quad 0 \le \theta < \frac{\pi}{2}$.

The tangent to *C* at the point *P* is perpendicular to the initial line.

a) Show that *P* has polar coordinates $\left(\frac{3}{2}, \frac{\pi}{6}\right)$.

The point *Q* on *C* has polar coordinates $\left(\sqrt{2}, \frac{\pi}{4}\right)$.

The shaded region *R* is bounded by *OP*, *OQ* and *C*, as shown in the figure above.

b) Show that the area of *R* is given by $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left(\sin^2 2\theta\cos 2\theta + \frac{1}{2} - \frac{1}{2}\cos 4\theta\right) \mathrm{d}\theta$.

c) Hence, or otherwise, find the area of *R*, giving your answer in the form $a + b\pi$, where *a* and *b* are rational numbers.

5. a) Given that $z = \cos\theta + \mathrm{i}\sin\theta$, use de Moivere's theorem to show that $z^n + \frac{1}{z^n} = 2\cos n\theta$.

b) Express $32\cos^6\theta$ in the form $p\cos 6\theta + q\cos 4\theta + r\cos 2\theta + s$, where *p*, *q*, *r* and *s* are integers.

c) Hence find the exact value of $\int_0^{\frac{\pi}{3}} \cos^6\theta \,\mathrm{d}\theta$.

6. You need to fit bins which are 10 units long, with items of length
3, 6, 2, 1, 5, 7, 2, 4, 1, 9 units respectively.
What is the minimum number of bins required to fit these items using

a) first fit

b) first fit decreasing

c) full bin combination

7. In the 80s Kim wished to video eight TV programmes.
The lengths of the programmes in minutes were
73 100 50 94 30 81 46 70

She decided to use 2 hour tapes (120 mins) to record all the programmes.

(a) Kim purchased four 2 hour tapes. Explain why she will not be able to fit her programmes on four tapes.
(b) Use a first fit decreasing bin- packing algorithm to find the minimum amount of tapes needed to record all programmes.
(c) Explain how you decided in which bin to place the programme of length 46.
(d) Was it possible for her to tape another two programmes of 20 mins without using another tape?
(e)

Questions 8-11 are:

Textbook questions Exam style paper p173
Questions 1,2,3,
And question 22 page 169

CHALLENGE QUESTION

Solve the inequalities

(i) $1 + 2x - x^2 > 2/x \qquad (x \neq 0)$,

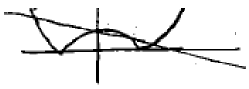
(ii) $\sqrt{(3x+10)} > 2 + \sqrt{(x+4)} \qquad (x \geqslant -10/3)$.

Answers:

Current work – M2 June 2006:

1)	6 s	2a)	14.4 kW	2b)	0.4m s^{-2}
3a)	25.1 Ns	3b)	18.9 m s^{-1}	4a)	i) $\frac{5}{2}a$
4a)	ii) $\frac{4}{3}a$	4b)	= 14.9°	5b)	3.5 s
7a)	22.4 J	7b)	6.4 m s^{-1}	7c)	4.27 m s^{-1}
8a)	$\left(\frac{1+e}{5}\right)u$, $\left(\frac{4e-1}{5}\right)u$	8c)	$\frac{3}{10}mu^2$		

Consolidation:

1a)	$K = 12$	1b)	$y = 2\cos 2x - \frac{\pi}{4}\sin 2x + 3x\sin 2x$	2a)	$x = 0, 1$
2b)		2c)	$x > \sqrt{7}-1$, $x < -\sqrt{7}-1$ and $0 < x < 1$	3)	$y = \frac{\sin x}{x} + \frac{\cos x}{x^2} + \frac{c}{x^2}$
4c)	$\frac{1}{6} + \frac{1}{24}\pi$	5b)	$\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$	5c)	$\frac{5}{48}\pi + \frac{3}{32}\sqrt{3}$

6. a) First fit 5 bins needed

1st bin	3, 6 1
2nd bin	2, 5, 2, 1
3rd bin	7
4th bin	4
5th bin	9

b) First fit decreasing 4 bins needed

1st bin	9, 1
2nd bin	7,3

3rd bin	6,4
4th bin	5,2,2,1

c) Full bin combinations 4 bins needed as in first fit or your own!

7. a) The total of the eight tv programmes is 544 minutes which is larger than 4x120 (=480)
b) In decreasing order the t v programmes are 100, 94, 81, 73, 70, 50, 46, 30

1st tape	100
2nd tape	94
3rd tape	81, 30
4th tape	73, 46
5th tape	70, 50

c) When it came to placing the 46 minute programme, the tape with most space available was the 4th one as it had 73 minutes already recorded.

d) Yes: they could go on Tapes 1 and 2

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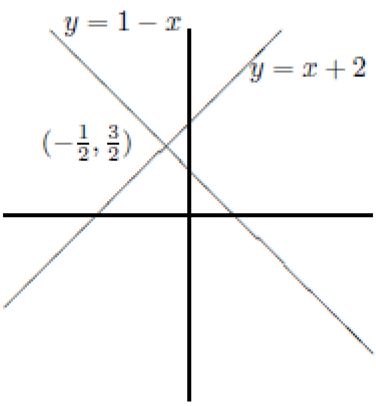
questions 8-11 answers in the textbook

Challenge question: tau

To do the multiplication, it pays to be systematic and to set out the algebra nicely:

$$\begin{aligned}(x-y+2)(x+y-1) &= x(x+y-1)-y(x+y-1)+2(x+y-1)\\ &= x^2+xy-x-yx-y^2+y+2x+2y-2\\ &= x^2+x-y^2+3y-2\end{aligned}$$

as required.



For the first inequality, we need $x-y+2$ and $x+y-1$ both to be either positive or negative. To sort out the inequalities (or inequations as they are sometimes horribly called), the first thing to do is to draw the lines corresponding to the corresponding equalities. These lines divide the plane into four regions and we then have to decide which regions are relevant.
The diagonal lines $x-y=-2$ and $x+y=1$ intersect at $(-\frac{1}{2},\frac{3}{2})$; this is the important point to mark in on the sketch. The required regions are the left and right quadrants formed by these diagonal lines (since the inequalities mean that the regions are to the right of both lines or to the left of both lines).

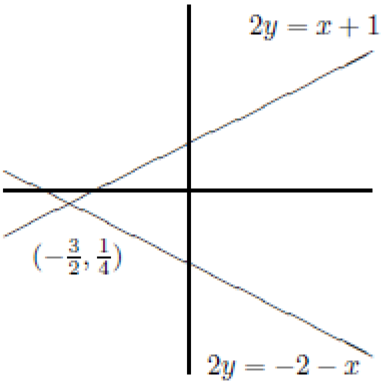
For the second part, the first thing to do is to factorise $x^2-4y^2+3x-2y+2$. The key similarity with the first part is the absence of 'cross' terms of the form xy . This allows a difference-of-two-squares factorisation of the first two terms: $x^2-4y^2=(x-2y)(x+2y)$. Following the pattern of the first part, we can then try a factorisation of the form

$$x^2-4y^2+3x-2y+2=(x-2y+a)(x+2y+b)$$

where $ab=2$. Considering the terms linear in x and y gives $a+b=3$ and $2a-2b=-2$ which quickly leads to $a=1$ and $b=2$. Note that altogether there were three equations for a and b, so we had no right to expect a consistent solution (except for the fact that this is a STEP question for which we had every right to believe that the first part would guide us through the second part).

Alternatively, we could have completed the square in x and in y and then used difference of two squares:

$$\begin{aligned}x^2-4y^2+3x-2y+2 &= (x+\tfrac{3}{2})^2-\tfrac{9}{4}-(2y+\tfrac{1}{2})^2+\tfrac{1}{4}+2\\ &= (x+\tfrac{3}{2})^2-(2y+\tfrac{1}{2})^2=(x+\tfrac{3}{2}-2y-\tfrac{1}{2})(x+\tfrac{3}{2}+2y+\tfrac{1}{2})\,.\end{aligned}$$



As in the previous case, the required area is formed by two intersecting lines; this time, they intersect at $(-\frac{3}{2},-\frac{1}{4})$ and the upper and lower regions are required, since the inequality is the other way round.

It is easy to see from the sketches that there are points that satisfy both inequalities: for example $(1,2)$.

BHASVIC

MαTHS

ASSIGNMENT COVER SHEET psi VJM adapted

Name -- Maths Teacher --

Question	Done	Backpack	Ready for test	Notes
M2 Edexcel Summer 2006				
Textbook questions				
1				
2				
3				
22				