α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	σ	τ	υ	**φ**	χ	ψ	ω

"The mathematician's patterns, like the painter's or the poet's, must be beautiful: the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test."

G H Hardy

Further Maths A2 (M2FP2D1) Assignment φ (phi) A due w/b 5th March 18

PREPARATION *Every week you will be required to do some preparation for future lessons, to be advised by your teacher.*

CURRENT WORK – MECHANICS

You need to complete the following past paper, timed and using good exam technique. Check the mark scheme only after you have completed the paper: **M2 Edexcel Summer 2005.**

CURRENT WORK – DECISION MATHS

To be advised by your teacher
Using the Decision Paper Solomon A select the questions you have covered in class and give them in with this assignment

CONSOLIDATION – FP2

1. a) Find the general solution of the differential equation $2\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+5\frac{\mathrm{d}x}{dt}+2x=2t+9$.

b) Find the particular solution of this differential equation for which $x = 3$ and $\frac{\mathrm{d}x}{\mathrm{d}t}=-1$ when $t = 0$.

The particular solution in part b) is used to model the motion of a particle P on the x-axis. At time t seconds $(t \geq 0)$, P is x metres from the origin O.

c) Show that the minimum distance between O and P is $\frac{1}{2}(5+\ln 2)$ m and justify that the distance is a minimum.

2. a) Show that the substitution $y = vx$ transforms the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3x-4y}{4x+3y} \qquad \text{(I)}$$

into the differential equation

$$x\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{3v^2+8v-3}{3v+4} \qquad \text{(II)}$$

b) By solving differential equation (II), find a general solution of differential equation (I)

c) Given that $y = 7$ at $x = 1$, show that the particular solution of differential equation (I) can be written as $(3y - x)(y + 3x) = 200$.

3. Solve the equation $z^5 = \mathrm{i}$, giving your answers in the form of $\cos\theta + \mathrm{i}\sin\theta$.

4. $(1+2x)\frac{\mathrm{d}y}{\mathrm{d}x}x + 4y^2 = 0$

a) Show that $(1+2x)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 1 + 2(4y-1)\frac{\mathrm{d}y}{\mathrm{d}x}$ (1)

b) Differentiate equation (1) with respect to x to obtain an equation involving $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}, \frac{\mathrm{d}^2y}{\mathrm{d}x^2}, \frac{\mathrm{d}y}{\mathrm{d}x}$, x and y.

Given that $y = \frac{1}{2}$ at $x = 0$,

c) find a series solution for y, in ascending powers of x, up to and including the term in x^3.

5. a) Find the Taylor expansion of cos $2x$ in ascending powers of $\left(x - \frac{\pi}{4}\right)$ up to and including the term in $\left(x - \frac{\pi}{4}\right)^5$.

b) Use your answer to a) to obtain an estimate of cos 2, giving your answer to 6 decimal places.

CHALLENGE QUESTION

Sketch the following subsets of the x-y plane:

(i) $|x| + |y| \leqslant 1$;

(ii) $|x-1| + |y-1| \leqslant 1$;

(iii) $|x-1| - |y+1| \leqslant 1$;

(iv) $|x|\,|y-2| \leqslant 1$.

Answers:
Current work:

1a)	35 m s^{-1}	1b)	14.6 or 15m s^{-1}	2a)	$\overline{x} = 3\text{cm}$
2b)	$\frac{1}{7}, 0.143$	3a)	$c = 4$	3b)	$\mathbf{a} = -36\mathbf{i} + 8\mathbf{j}$
4a)	x=1.8m	4b)	6.75 or 6.8	5a)	$\frac{2}{3}$

$-\frac{2}{7}u$

5b) $\frac{2}{7}u < \frac{2}{3}u$ so B does not overtake A

So no more collisions

6a) 1020 N

6b) 778 N or 780N

6c) In equilibrium all forces act through a point P and weight meet at mid-point; hence reaction also acts through mid-point so reaction horizontal
OR M(mid-point): $Y \times 1.5 = 0 \Rightarrow Y = 0$
Hence reaction is horizontal

7a) 118J or 120J

7b) 10 N

7c) 0.393 or 0.39 or 0.4

7d) 5.39 or 5.4 m s^{-1}

Consolidation

1a) $x = Ae^{-2t} + Be^{-\frac{1}{2}t} + t + 2$

1b) $x = e^{-2t} + t + 2$

2b) $\frac{1}{2}\ln\left(\frac{3y^2}{x^2} + \frac{8y}{x} - 3\right) = -\ln x + C$

3) $\cos\frac{9\pi}{10} + i\sin\frac{9\pi}{10}$
$\cos\frac{13\pi}{10} + i\sin\frac{13\pi}{10}$
$\cos\frac{17\pi}{10} + i\sin\frac{17\pi}{10}$

4b) $(1+2x)\frac{d^3y}{dx^3} = 8\left(\frac{dy}{dx}\right)^2 + 4(2y-1)\frac{d^2y}{dx^2}$

4c) $y = \frac{1}{2} + x + \frac{3}{2}x^2 + \frac{4}{3}x^3 + \ldots$

5a) $-2(x-\frac{\pi}{4}) + \frac{4}{3}(x-\frac{\pi}{4})^3 - \frac{4}{15}(x-\frac{\pi}{4})^5 + \ldots$

5b) $-0.416147\ldots$

Challenge question: rho

Only integers ending in 1, 3, 7, or 9 are not divisible by 2 or 5. This is 4/10 of the possible integers, so the total number of such integers is 4/10 of 1,000, i.e. 400.

The integers can be added in pairs:

$$\text{sum} = (1 + 999) + (3 + 997) + \cdots + (499 + 501) .$$

There are 200 such pairs, so the sum is $1,000 \times 200$ and the average is 500. A simpler argument would be to say that this is obvious by symmetry: there is nothing in the problem that favours an answer greater (or smaller) than 500.

Alternatively, we can find the number of integers divisible by both 2 and 5 by adding the number divisible by 2 (i.e. 500) to the number divisible by 5 (i.e. 200), and subtracting the number divisible by both 2 and 5 (i.e. 100) since these have been counted twice. To find the sum, we can sum those divisible by 2 (using the formula for a geometric progression), add the sum of those divisible by 5 and subtract the sum of those divisible by 10.

For the second part, either of these methods will work. In the first method you consider integers in blocks of 21 (essentially arithmetic to base 21): there are 12 integers in each such block that are not divisible by 3 or 7 (namely 1, 2, 4, 5, 8, 10, 11, 13, 16, 17, 19, 20) so the total number is $9261 \times 12/21 = 5292$. The average is 9261/2 as can be seen using the pairing argument $(1 + 9260) + (2 + 9259) + \cdots$ or the symmetry argument.

BHASVIC
MaTHS

ASSIGNMENT COVER SHEET phi

Name ______________________ Maths Teacher ______________________

Question	Done	Backpack	Ready for test	Notes
M2 Edexcel Summer 2005				
Decision Solomon A				
1				
2				
3				
4				
5				