α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	σ	τ	υ	φ	χ	ψ	ω

"The mathematician's patterns, like the painter's or the poet's, must be beautiful: the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test." G H Hardy

Further Maths A2 (M2FP2D1) Assignment υ (upsilon) A
Due w/b 26th Feb 18

5 questions and an M2 paper to be given in

PREPARATION *Every week you will be required to do some preparation for future lessons, to be advised by your teacher.*

CURRENT WORK – MECHANICS

You need to complete the following past paper, timed and using good exam technique. Check the mark scheme only after you have completed the paper: **M2 Edexcel January 2005.**

CONSOLIDATION – FP2

1. a) Show that the transformation $y = xv$ transforms the equation

$$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2x\frac{\mathrm{d}y}{\mathrm{d}x} + \left(2 + 9x^2\right)y = x^5 \qquad \text{(I)}$$

into the equation

$$\frac{\mathrm{d}^2v}{\mathrm{d}x^2} + 9v = x^2 \qquad \text{(II)}$$

b) Solve the differential equation II to find v as a function of x.

c) Hence state the general solution of the differential equation I.

2. Find the general solution of the differential equation $(x+1)\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = \frac{1}{x}, \quad x > 0$ giving your answer in the form $y = \mathrm{f}(x)$.

3. a) Given that $z = \mathrm{e}^{1\theta}$, show that $z^n - \frac{1}{z^n} = 2\mathrm{i}\sin n\theta$, where n is a positive integer.

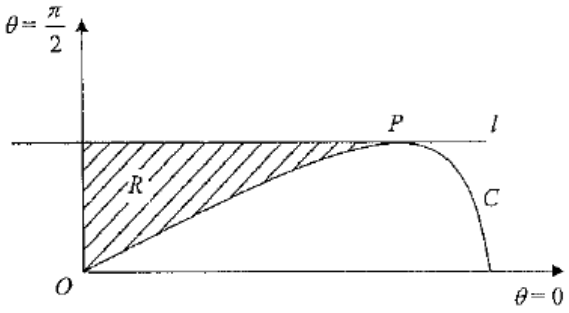
b) Show that $\sin^5\theta = \frac{1}{16}\left(\sin 5\theta - 5\sin 3\theta + 10\sin\theta\right)$.

c) Hence solve, in the interval $0 \le \theta < 2\pi$, $\sin 5\theta - 5\sin 3\theta + 6\sin\theta = 0$

4. Find the set of values of x for which

$$\frac{x^2}{x-2} > 2x \ .$$

5.



A curve C has polar equation $r^2 = a^2 \cos 2\theta, 0 \le \theta \le \frac{\pi}{4}$. The line l is parallel to the initial line, and l is the tangent to C at the point P, as shown in the figure above.

a) (i) Show that, for any point on C, $r^2 \sin^2 \theta$ can be expressed in terms of $\sin\theta$ and a only.

(ii) Hence, using differentiation, show that the polar coordinates of P are $\left(\frac{a}{\sqrt{2}}, \frac{\pi}{6}\right)$.

The shaded region R, shown in the figure above, is bounded by C, the line l and the half-line with equation $\theta = \frac{\pi}{2}$.

b) Show that the area of R is $\frac{a^2}{16}\left(3\sqrt{3}-4\right)$.

CHALLENGE QUESTION

How many integers greater than or equal to zero and less than 1000 are not divisible by 2 or 5? What is the average value of these integers?

How many integers greater than or equal to zero and less than 9261 are not divisible by 3 or 7? What is the average value of these integers?

Answers:

Current work:

1b)	$\frac{2}{3}$ W	2a)	10.7 cm	3b)	$\theta = 25°$
3a)	41.0 J	3b)	0.67	4a)	5.0
4b)	78 m	5b)	1.4 m s^{-2}	5c)	850 N
5d)	335 kJ	5e)	Resistances *vary* with *speeds*	6b)	$\frac{2}{3} < e \leq 1$
6c)	$e = \frac{7}{9}$	7a)	4.77 s	7b)	122 m
7c)	33.2 m s^{-1}	7d)	39.6°		

Consolidation

1b)	$v = A\sin 3x + B\cos 3x + \frac{1}{9}x^2 - \frac{2}{81}$	1c)	$y = Ax\sin 3x + Bx\cos 3x + \frac{1}{9}x^3 - \frac{2}{81}x$	2	$y = (1+x)^{-2}(x + \ln x + c)$
3c)	$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$	4)	$2 < x < 4$		

Challenge question: pi

First we expand binomially:

$$\begin{aligned}
&\left(1 - \frac{2}{100}\right)^{\frac{1}{2}} \\
\approx\ & 1 + \left(\frac{1}{2}\right)\left(-\frac{2}{100}\right) + \left(\frac{1}{2!}\right)\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{2}{100}\right)^2 - \left(\frac{1}{3!}\right)\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{2}{100}\right)^3 + \cdots \\
=\ & 1 - \frac{1}{100} - \frac{0.5}{10^4} - \frac{0.5}{10^6} = 0.9899495\,.
\end{aligned}$$

It is clear that the next term in the expansion would introduce the seventh and eighth places of decimals, which it seems we do not need. Of course, after further manipulations we might find that the above calculation does not supply the 6 decimal places we need, in which case we will work out the next term in the expansion.

But $\left(\frac{98}{100}\right)^{\frac{1}{2}} = \frac{7\sqrt{2}}{10}$, so $\sqrt{2} \approx 9.899495/7 \approx 1.414214\,.$

Second part:

$$\begin{aligned}
&\left(1 + \frac{3}{125}\right)^{\frac{1}{3}} \\
\approx\ & 1 + \left(\frac{1}{3}\right)\left(\frac{3}{125}\right) + \left(\frac{1}{2!}\right)\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(\frac{3}{125}\right)^2 + \left(\frac{1}{3!}\right)\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)\left(-\frac{3}{125}\right)^3 + \cdots \\
=\ & 1 - \frac{8}{1000} - \frac{64}{10^6} + \frac{5}{3}\frac{8^3}{10^9} \\
=\ & 1.007936 + \frac{256}{3}\frac{1}{10^8} = 1.007937\,.
\end{aligned}$$

Successive terms in the expansion decrease by a factor of about 1000, so this should give the right number of decimal places.

But $\left(\frac{128}{125}\right)^{\frac{1}{3}} = \frac{4\sqrt[3]{2}}{5} = \frac{8\sqrt[3]{2}}{10}$ so $\sqrt[3]{2} \approx 10.0793/8 \approx 1.259921\,.$

BHASVIC MATHS

ASSIGNMENT COVER SHEET upsilon

Name ..... Maths Teacher .....

Question	Done	Backpack	Ready for test	Notes
M2 Edexcel January 2005				
1				
2				
3				
4				
5				