

|          |         |          |          |               |         |        |          |         |          |           |       |       |       |          |       |        |          |        |       |           |        |        |          |
|----------|---------|----------|----------|---------------|---------|--------|----------|---------|----------|-----------|-------|-------|-------|----------|-------|--------|----------|--------|-------|-----------|--------|--------|----------|
| $\alpha$ | $\beta$ | $\gamma$ | $\delta$ | $\varepsilon$ | $\zeta$ | $\eta$ | $\theta$ | $\iota$ | $\kappa$ | $\lambda$ | $\mu$ | $\nu$ | $\xi$ | $\sigma$ | $\pi$ | $\rho$ | $\sigma$ | $\tau$ | $\nu$ | $\varphi$ | $\chi$ | $\psi$ | $\omega$ |
|----------|---------|----------|----------|---------------|---------|--------|----------|---------|----------|-----------|-------|-------|-------|----------|-------|--------|----------|--------|-------|-----------|--------|--------|----------|

“A mathematician is a machine for turning coffee into theorems”

P Erdos

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Come along to the Maths Association Talk – 29<sup>th</sup> September at 4:30pm in room 3

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## Further Maths A2 (M2FP2D1) Assignment $\alpha$ (alpha) A

| Test                | Application date | Test date | Universities interested | Compulsory?  |
|---------------------|------------------|-----------|-------------------------|--|
| Test of Mathematics | 15/10            | 9/11      | Lancaster<br>Durham     | No but may result in reduced offer   |
| MAT                 | 15/10            | 2/11      | Oxford                  | Yes  |
| MAT                 | 15/10            | 2/11      | Imperial                | Expected<br>But if you apply after the 15/10 then you will have to do a STEP paper |

**DRILL** Drills are the very basic techniques you need to solve maths problems. You need to practise these until you can do them quickly and accurately. Answers are not provided for drill questions.

**A.** Find:

$$1) \quad \frac{1}{2} \int r^2 d\theta \qquad 2) \quad \frac{d}{d\theta}(r \sin \theta) \qquad 3) \quad \frac{d}{d\theta}(r \cos \theta)$$

Given:  $r = 3 \sin 2\theta$

**B.** Find:

$$1) \quad \frac{1}{2} \int r^2 d\theta \qquad 2) \quad \frac{d}{d\theta}(r \sin \theta) \qquad 3) \quad \frac{d}{d\theta}(r \cos \theta)$$

Given:  $r = 4 + \cos 2\theta$

**PREPARATION** Every week you will be required to do some preparation for future lessons, to be advised by your teacher.

# CURRENT WORK AND CONSOLIDATION

FP2 inequalities

1. Solve each of these inequalities

$$(a) \frac{2x-1}{x-2} \geq 0$$

$$(b) \frac{3x-2}{2-x} > 0$$

$$(c) \frac{2x^2+x-1}{x^2-1} \leq 0$$

$$(d) \frac{1-2x}{3-x} > 2$$

$$(e) \frac{x+a}{x-2} < 0$$

$$(f) \frac{x}{(x-2)(x+3)} > 0$$

$$(g) \frac{4x+1}{x} > -4x$$

$$(h) |2x^2 - 3x - 2| = 3(x+1)$$

2. (a) Show that  $r = \frac{1}{2}(r(r+1) - r(r-1))$

$$(b) \text{ Hence show that } \sum_{r=1}^n r = \frac{n}{2}(n+1)$$

3. Given  $\frac{1}{r(r+1)(r+2)} \equiv \frac{1}{2r(r+1)} - \frac{1}{2(r+1)(r+2)}$

find  $\sum_{r=1}^n \frac{1}{r(r+1)(r+2)}$  using the method of differences

4. (a) Express  $\frac{1}{r(r+2)}$  in partial fractions

$$(b) \text{ Hence find the sum of the series } \sum_{r=1}^n \frac{1}{r(r+2)} \text{ using the method of differences}$$

5. (a) Express  $\frac{1}{(r+2)(r+3)}$  in partial fractions

$$(b) \text{ Hence find the sum of the series } \sum_{r=1}^n \frac{1}{(r+2)(r+3)} \text{ using the method of differences}$$

6. Use the identity  $4r^3 \equiv r^2(r+1)^2 - (r-1)^2 r^2$

to find  $\sum_{r=1}^n r^3$

7. Given that  $\frac{r}{(r+1)!} \equiv \frac{1}{r!} - \frac{1}{(r+1)!}$

find  $\sum_{r=1}^n \frac{r}{(r+1)!}$

8. Given that  $\frac{2r+1}{r^2(r+1)^2} \equiv \frac{1}{r^2} - \frac{1}{(r+1)^2}$

$$\text{find } \sum_{r=1}^n \frac{2r+1}{r^2(r+1)^2}$$

Solve the following differential equations

9.

$$x \frac{dy}{dx} + y = e^x$$

10.

$$\frac{x}{y} \frac{dy}{dx} + \ln y = x + 1$$

11.

$$e^x y + e^x \frac{dy}{dx} = 2$$

12.

$$\frac{dy}{dx} + 3y = e^{-3x}$$

13.

$$x^2 \frac{dy}{dx} + xy = x + 1$$

14.

$$\tan x \frac{dy}{dx} + y = e^x \tan x$$

15. Given that  $y = 0$  when  $x = \frac{\pi}{2}$ , solve

$$x \frac{dy}{dx} = y + x^2(\sin x + x \cos x)$$

16 Find the general solution of the following differential equations:

(a)  $\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 3y = 12$

(b)  $\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + 2y = 4x$

(c)  $\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = e^{2x}$

(d)  $\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 4y = 2x - 1$

(e)  $\frac{d^2y}{dx^2} + y = \cos 2x$

(f)  $\frac{d^2y}{dx^2} + 9y = e^{\frac{1}{2}x}$

17. Find the solution subject to the given boundary conditions for each of the following differential equations:

(a)  $\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 3y = 12; \frac{dy}{dx} = 1 \text{ and } y = 0 \text{ at } x = 0$

(b)  $\frac{d^2y}{dx^2} + y = e^x; \frac{dy}{dx} = y = 0 \text{ at } x = 0$

(c)  $\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = \cos x; \frac{dy}{dx} = 0 \text{ and } y = 1 \text{ at } x = 0$

(d)  $\frac{d^2y}{dx^2} - 6 \frac{dy}{dx} + 5y = e^{2x}; \frac{dy}{dx} = y = 2 \text{ at } x = 0$

(e)  $\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 2y = 4x; \frac{dy}{dx} = y = 0 \text{ at } x = 0$

(f)  $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 2x + 4; y = 1, \frac{dy}{dx} = 0 \text{ at } x = 0$

**Answers:**

(1a)  $x \leq \frac{1}{2}, x > 2$

(1b)  $\frac{2}{3} < x < 2$

(1c)  $\frac{1}{2} \leq x < 1$

(1d)  $x > 3$

(1e)  $-a < x < 2$

(1f)  $-3 < x < 0, x > 2$

(1g)  $x > 0$

(1h)  $x = \frac{1}{2}(3 \pm \sqrt{19})$

(2a) Discuss in class

(2b) Discuss in class

(3)  $\frac{n(n+3)}{4(n+1)(n+2)}$

(4a)  $\frac{1}{2r} - \frac{1}{(r+2)}$

(4b)  $\frac{n(3n+5)}{4(n+1)(n+2)}$

(5a)  $\frac{1}{(r+2)(r+3)} = \frac{1}{r+2} - \frac{1}{r+3}$

(5b)  $\frac{n}{3(n+3)}$

(6)  $\frac{2}{r} + \frac{1}{r+1} - \frac{3}{r+2}$

(7) Discuss in class

(8)  $\frac{n(n+2)}{(n+1)^2}$

9)  $xy = e^x + A$

10)  $2x \ln y = x^2 + 2x + A$

11)  $ye^x = 2x + A$

12)  $ye^{3x} = x + A$

13)  $xy = x + \ln Ax$

14)  $2y \sin x = e^x(\sin x - \cos x) + A$

15)  $y = x(\sin x - \cos x - 1)$

16a)  $y = Ae^x + Be^{3x} + 4$

(16b)  $y = Ae^{-x} + Be^{-2x} + 2x - 3$

(16c)  $y = (A + Bx)e^x + e^{2x}$

(16d)  $y = (A + Bx)e^{-2x} + \frac{1}{2}x - \frac{3}{4}$

(16e)  $y = A \cos x + B \sin x - \frac{1}{3} \cos 2x$

(16f)  $y = A \cos 3x + B \sin 3x + \frac{4}{37}e^{\frac{1}{2}x}$

(17a)  $y = -\frac{13}{2}e^x + \frac{5}{2}e^{3x} + 4$

(17b)  $y = -\frac{1}{2}(\cos x + \sin x - e^x)$

(17c)  $y = (1 - \frac{1}{2}x)e^x - \frac{1}{2}\sin x$

(17d)  $y = \frac{9}{4}e^x + \frac{1}{12}e^{5x} - \frac{1}{3}e^{2x}$

(17e)  $y = 2e^{-x} \cos x + 2x - 2$

(17f)  $y = \frac{1}{2}(e^{-2x} + 1)(x + 1)$

# BHASVIC

# MATHS

## ASSIGNMENT

## COVER SHEET

Name \_\_\_\_\_ Maths Teacher \_\_\_\_\_

| Question | Done | Backpack | Ready for test | Notes |
|----------|------|----------|----------------|-------|
| Drill A  |      |          |                |       |
| Drill B  |      |          |                |       |
| 1        |      |          |                |       |
| 2        |      |          |                |       |
| 3        |      |          |                |       |
| 4        |      |          |                |       |
| 5        |      |          |                |       |
| 6        |      |          |                |       |
| 7        |      |          |                |       |
| 8        |      |          |                |       |
| 9        |      |          |                |       |
| 10       |      |          |                |       |
| 11       |      |          |                |       |
| 12       |      |          |                |       |
| 13       |      |          |                |       |
| 14       |      |          |                |       |
| 15       |      |          |                |       |
| 16       |      |          |                |       |
| 17       |      |          |                |       |