α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	σ	$\boxed{\tau}$	υ	φ	χ	ψ	ω

"The mathematician's patterns, like the painter's or the poet's, must be beautiful: the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test." G H Hardy

Further Maths A2 (M2FP2D1) Assignment τ (tau) A
Due in w/b 19th Feb 18

PREPARATION *Every week you will be required to do some preparation for future lessons, to be advised by your teacher.*

CURRENT WORK – MECHANICS

1. A car of mass 1000 kg is towing a trailer of mass 1500 kg along a straight horizontal road. The two-bar joining the car to the trailer is modelled as a light rod parallel to the road. The total resistance to motion of the car is modelled has having constant magnitude 750 N. The total resistance to motion of the trailer is modelled as a force of magnitude R newtons, where R is a constant. When the engine is working at a rate of 50 kW, the car and the trailer travel at a constant speed of 25 m s^{-1}.

   a) Show that $R = 1250$

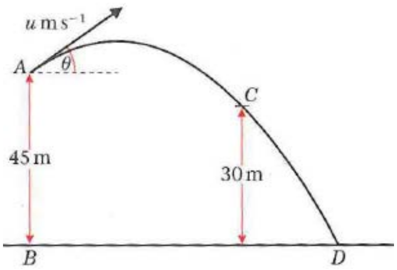
   When travelling at 25 m s^{-1} the driver of the car disengages the engine and applies the brakes. The brakes provide a constant braking force of magnitude 1500 N to the car. The resisting forces of magnitude 750 N and 1250 N are assumed to remain unchanged. Calculate

   b) the deceleration of the car while breaking,

   c) the thrust in the two-bar while braking,

   d) the work done, in kJ, by the braking force in bringing the car and the trailer to rest.

   e) Suggest how the modelling assumption that the resistances to motion are constant could be refined to be more realistic.

2. 

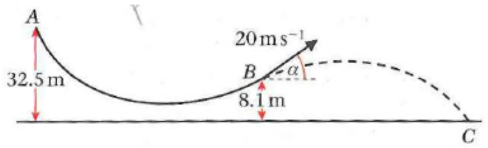
   A particle P is projected from a point A with speed u m s^{-1} at an angle of elevation θ, where $\cos\theta = \frac{4}{5}$. The point B, on horizontal ground, is vertically below A and AB = 45 m. After projection, P moves freely under gravity passing through a point C, 30 m above the horizontal ground, before striking the ground at the point D, as shown in the figure above.

   Given that P passes through C with speed 24.5 m s^{-1},

   a) using conservation of energy, or otherwise, show that $u = 17.5$,

   b) find the size of the angle which the velocity of P makes with the horizontal as P passes through C,

   c) find the distance BD.

3. 

In a ski-jump competition, a skier of mass 80 kg moves from rest at a point A on a ski-slope. The skier's path is an arc AB. The starting point A of the slope is 32.5 m above horizontal ground. The end B of the slope is 8.1 m above the ground. When the skier reaches B she is travelling at 20 m s^{-1} and moving upwards at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$, as shown in the figure. The distance along the slope from A to B is 60 m. The resistance to motion while she is on the slope is modelled as a force of constant magnitude R newtons.

a) By using the work-energy principle, find the value of R.

On reaching B, the skier then moves through the air and reaches the ground at the point C. The motion of the skier in moving from B to C is modelled as that of a particle moving freely under gravity.

b) Find the time the skier takes to move from B to C.

c) Find the horizontal distance from B to C.

d) Find the speed of the skier immediately before she reaches C.

4. A particle P of mass 0.75 kg is moving under the action of a single force **F** newtons. At time t seconds, the velocity **v** m s^{-1} of P is given by $\mathbf{v} = \left(t^2 + 2\right)\mathbf{i} - 6t\mathbf{j}$.

a) Find the magnitude of **F** when $t = 4$.

When $t = 5$, the particle P receives an impulse of magnitude $9\sqrt{2}$ Ns in the direction of the vector $\mathbf{i} - \mathbf{j}$.

b) Find the velocity of P immediately after the impulse.

5. A tennis ball of mass 0.2 kg is moving with velocity $(-10\mathbf{i})$ m s^{-1} when it is struck by a tennis racket. Immediately after being struck, the ball has velocity $(15\mathbf{i} + 15\mathbf{j})$ m s^{-1}. Find

a) the magnitude of the impulse exerted by the racket on the ball,

b) the angle, to the nearest degree, between the vector **i** and the impulse exerted by the racket,

c) the kinetic energy gained by the ball as a result of being struck.

6. The unit vectors **i** and **j** lie in a vertical plane, **i** being horizontal and **j** vertical. A ball of mass 0.1 kg is hit by a bat which gives it an impulse of $(3.5\mathbf{i} + 3\mathbf{j})$ Ns. The velocity of the ball immediately after being hit is $(10\mathbf{i} + 25\mathbf{j})$ m s^{-1}.

a) Find the velocity of the ball immediately before it is hit.

In the subsequent motion the ball is modelled as a particle moving freely under gravity. When it is hit the ball is 1 m above horizontal ground.

b) Find the greatest height of the all above the ground in the subsequent motion.

The ball is caught when it is again 1 m above the ground.

c) Find the distance from the point where the ball is hit to the point where it is caught.

7. A particle A of mass $2m$, moving with speed $2u$ in a straight line on a smooth horizontal table, collides

with a particle B of mass $3m$, moving with speed u in the same direction as A. The coefficient of restitution between A and B is e.

a) Show that the speed of B after the collision is $\frac{1}{5}u(7+2e)$.

b) Find the speed of A after the collision, in terms of u and e.

The speed of A after the collision is $\frac{11}{10}u$.

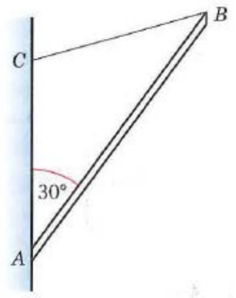
c) Show that $e=\frac{1}{2}$.

At the instant of collision, A and B are at a distance d from a vertical barrier fixed to the surface at right-angles to their direction of motion. Given that B hits the barrier, and that the coefficient of restitution between B and the barrier is $\frac{11}{16}$,

d) find the distance of A from the barrier at the instant that B hits the barrier,

e) show that, after B rebounds from the barrier, it collides with A again at a distance $\frac{5}{32}d$ from the barrier.

8.



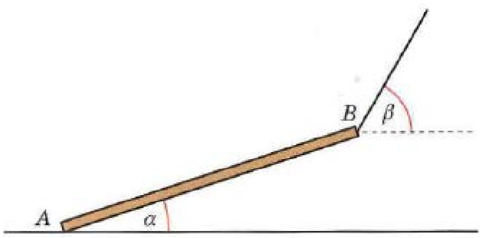
A uniform rod AB of mass m and length $2a$ is smoothly hinged to a vertical wall at A and is supported in equilibrium by a rope which is modelled as a light string. One end of the rope is attached to the end B of the rod and the other end is attached to a point C of the wall, where C is vertically above A, $AC = CB$, and $\angle CAB = 30°$, as shown in the diagram.

a) Show that the tension in the rope is $0.5mg$.

b) Find the magnitude of the vertical component of the force acting on the rod at A.

c) If the rope were not modelled as a light string, state how this would affect the tension throughout the rope.

9.



A straight log AB has weight W and length $2a$. A cable is attached to one end B of the log. The cable lifts the end B off the ground. The end A remains in contact with the ground, which is rough and horizontal. The log is in limited equilibrium. The log makes an angle α to the horizontal, where $\tan\alpha=\frac{5}{12}$. The cable makes an angle β to the horizontal, as shown in the diagram. The coefficient of friction between the log and the ground is 0.6. the log is modelled as a uniform rod and the cable as light.

a) Show that the normal reaction on the log at A is $\frac{2}{5}W$.

b) Find the value of β.

The tension in the cable is kW.

c) Find the value of k.

CONSOLIDATION – FP2

10. a) Find the general solution of the differential equation

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+2y=2e^{-t}$$

b)Find the particular solution that satisfies $y = 1$ and $\frac{dy}{dt}=1$ at $t = 0$.

11. Find the general solution of the differential equation

$$\frac{dy}{dx}+2y\cot 2x=\sin x, \quad 0<x<\frac{\pi}{2},$$

giving your answer in the form $y = f(x)$.

12. a) By expressing $\frac{2}{4r^2-1}$ in partial fractions, or otherwise, prove that $\sum_{r=1}^{n}\frac{2}{4r^2-1}=1-\frac{1}{2n+1}$.

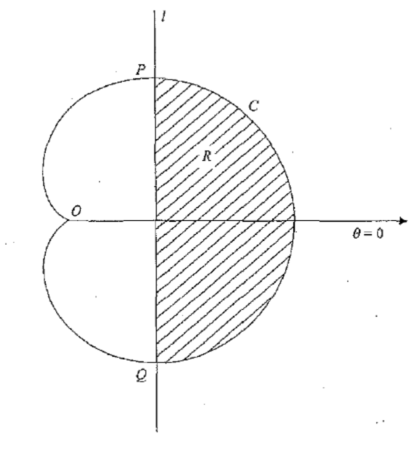
b) Hence find the exact value of $\sum_{r=11}^{20}\frac{2}{4r^2-1}$.

13. a) On the same diagram, sketch the graphs of $y=\left|x^2-4\right|$ and $y=\left|2x-1\right|$, showing the coordinates of the points where the graphs meet the axes.

b) Solve $\left|x^2-4\right|=\left|2x-1\right|$, giving your answers in surd form where appropriate.

c) Hence, or otherwise, find the set of values of x for which of $\left|x^2-4\right|>\left|2x-1\right|$.

14.



The curve C which passes through O has polar equation $r=4a\left(1+\cos\theta\right), \quad -\pi<\theta\le\pi$.

The line l has polar equation $r=3a\sec\theta, \quad -\frac{\pi}{2}<\theta<\frac{\pi}{2}$.

The line l cuts C at points P and Q, as shown in the figure above.

a) Prove that $PQ = 6\sqrt{3}a$.

The region R, shown shaded in the figure above, is bounded by l and C.

b) Use calculus to find the exact area of R.

15. The point P represents a complex number z on an Argand diagram, where $|z-6+3\mathrm{i}| = 3|z+2-\mathrm{i}|$.

a) Show that the locus of P is a circle, giving the coordinates of the centre and the radius of this circle.

The point Q represents a complex number z on an Argand diagram, where $\tan\left[\arg(z+6)\right] = \frac{1}{2}$.

b) On the same Argand diagram, sketch the locus of P and the locus of Q.

c) On your diagram, shade the region which satisfies both $|z-6+3\mathrm{i}| = 3|z+2-\mathrm{i}|$ and $\tan\left[\arg(z+6)\right] = \frac{1}{2}$.

CHALLENGE QUESTION

Use the first four terms of the binomial expansion of $(1 - 1/50)^{1/2}$, to derive the approximation $\sqrt{2} \approx 1.414214$.

Calculate similarly an approximation to the cube root of 2 to six decimal places by considering $(1 + N/125)^{1/3}$, where N is a suitable number.

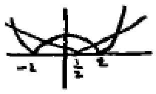
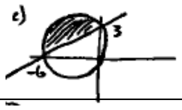
[You need not justify the accuracy of your approximations.]

Answers:

Current work:

1b)	$1.4\ \mathrm{m\ s^{-2}}$	1c)	850 N	1d)	335 kJ (3 s.f.)
1e)	The resistance could be modelled as varying with speed.			2b)	$\psi = 55°$ (nearest degree)
2c)	60 m	3a)	52 (2 s.f.)	3b)	3 s
3c)	48 m	3d)	$24\ \mathrm{m\ s^{-1}}$ (2 s.f.)	4a)	7.5 N
4b)	$(39\mathbf{i} - 42\mathbf{j})\ \mathrm{m\ s^{-1}}$	5a)	5.83 Ns	5b)	31° (nearest degree)
5c)	35 J	6a)	$(-25\mathbf{i} - 5\mathbf{j})\ \mathrm{m\ s^{-1}}$	6b)	32.9 m
6c)	51 m	7b)	$v_A = \frac{1}{5}u(7-3e)$	7d)	$\frac{5d}{16}$
8b)	$Y = \dfrac{5mg}{4}$	8c)	The tension would not be constant throughout the length of the string.		
9b)	68.2°	9c)	0.646		

Consolidation

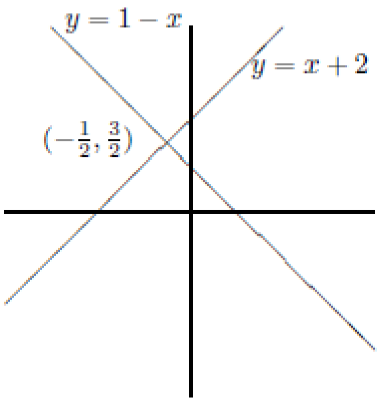
10a)	$y = e^{-t}(A\cos t + B\sin t + 2)$	10b)	$y = e^{-t}(2\sin t - \cos t + 2)$	11)	$y = \dfrac{2\sin^3 x}{3\sin 2x} + \dfrac{c}{\sin 2x}$
12b)	$\frac{10}{861}$	13a)		13b)	$x = -1 \pm \sqrt{6}$
13c)	$-1 < x < \sqrt{6} - 1,\quad x > \sqrt{3}$	14b)	$a^2(8\pi + 9\sqrt{3})$	15a)	centre $(-3, \frac{3}{2})$ radius $\frac{3}{2}\sqrt{5}$
15b)					

Challenge question: tau

To do the multiplication, it pays to be systematic and to set out the algebra nicely:

$$\begin{aligned}(x-y+2)(x+y-1) &= x(x+y-1)-y(x+y-1)+2(x+y-1)\\ &= x^2+xy-x-yx-y^2+y+2x+2y-2\\ &= x^2+x-y^2+3y-2\end{aligned}$$

as required.



For the first inequality, we need $x-y+2$ and $x+y-1$ both to be either positive or negative. To sort out the inequalities (or inequations as they are sometimes horribly called), the first thing to do is to draw the lines corresponding to the corresponding equalities. These lines divide the plane into four regions and we then have to decide which regions are relevant.

The diagonal lines $x-y=-2$ and $x+y=1$ intersect at $(-\frac{1}{2},\frac{3}{2})$; this is the important point to mark in on the sketch. The required regions are the left and right quadrants formed by these diagonal lines (since the inequalities mean that the regions are to the right of both lines or to the left of both lines).

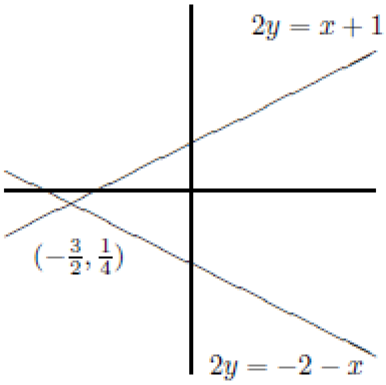
For the second part, the first thing to do is to factorise $x^2-4y^2+3x-2y+2$. The key similarity with the first part is the absence of 'cross' terms of the form xy. This allows a difference-of-two-squares factorisation of the first two terms: $x^2-4y^2=(x-2y)(x+2y)$. Following the pattern of the first part, we can then try a factorisation of the form

$$x^2-4y^2+3x-2y+2=(x-2y+a)(x+2y+b)$$

where $ab=2$. Considering the terms linear in x and y gives $a+b=3$ and $2a-2b=-2$ which quickly leads to $a=1$ and $b=2$. Note that altogether there were three equations for a and b, so we had no right to expect a consistent solution (except for the fact that this is a STEP question for which we had every right to believe that the first part would guide us through the second part).

Alternatively, we could have completed the square in x and in y and then used difference of two squares:

$$\begin{aligned}x^2-4y^2+3x-2y+2 &= (x+\tfrac{3}{2})^2-\tfrac{9}{4}-(2y+\tfrac{1}{2})^2+\tfrac{1}{4}+2\\ &= (x+\tfrac{3}{2})^2-(2y+\tfrac{1}{2})^2=(x+\tfrac{3}{2}-2y-\tfrac{1}{2})(x+\tfrac{3}{2}+2y+\tfrac{1}{2})\,.\end{aligned}$$



As in the previous case, the required area is formed by two intersecting lines; this time, they intersect at $(-\frac{3}{2},-\frac{1}{4})$ and the upper and lower regions are required, since the inequality is the other way round.

It is easy to see from the sketches that there are points that satisfy both inequalities: for example $(1,2)$.

BHASVIC

MαTHS

ASSIGNMENT COVER SHEET tau

Name -- Maths Teacher --

Question	Done	Backpack	Ready for test	Notes
1				
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