α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	$\boxed{\sigma}$	τ	υ	φ	χ	ψ	ω

"The mathematician's patterns, like the painter's or the poet's, must be beautiful: the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test."

G H Hardy

Further Maths A2 (M2FP2D1) Assignment σ (sigma) A
Due 5th Feb 18

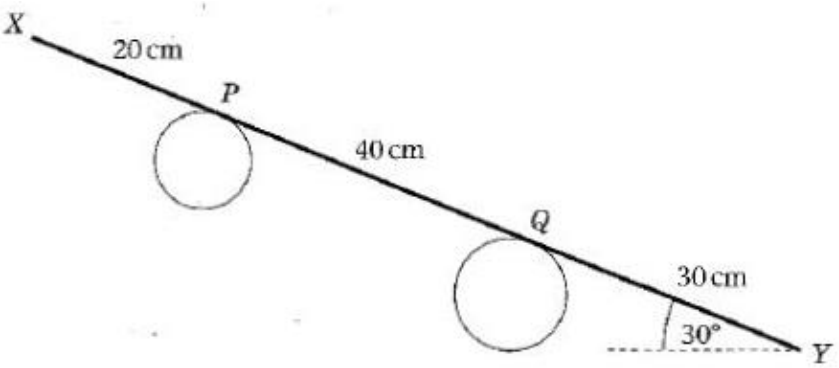
PREPARATION *Every week you will be required to do some preparation for future lessons, to be advised by your teacher.*

CURRENT WORK – MECHANICS

1. A uniform ladder AB of mass 20 kg rests with its top A against a smooth vertical wall and its base B on rough horizontal ground. The coefficient of friction between the ladder and the ground is $\frac{3}{4}$. A mass of 10 kg is attached to the ladder. Given that the ladder is about to slip, find the inclination of the ladder to the horizontal

   a if the 10 kg mass is attached at A,

   b if the 10 kg mass is attached at B.

2. 

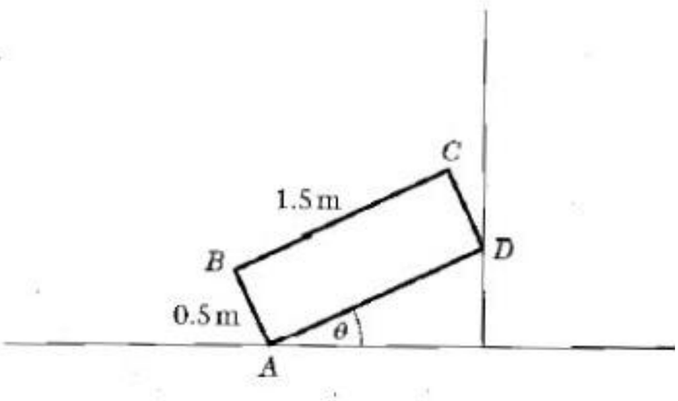
   A uniform rod XY has weight 20 N and length 90 cm. The rod rests on two parallel pegs, with X above Y, in a vertical plane which is perpendicular to the axes of the pegs, as shown in the diagram. The rod makes an angle of 30° to the horizontal and touches the two pegs at P and Q, where $XP = 20$ cm and $XQ = 60$ cm.

   a Calculate the normal components of the forces on the rod at P and at Q.

   The coefficient of friction between the rod and each peg is μ.

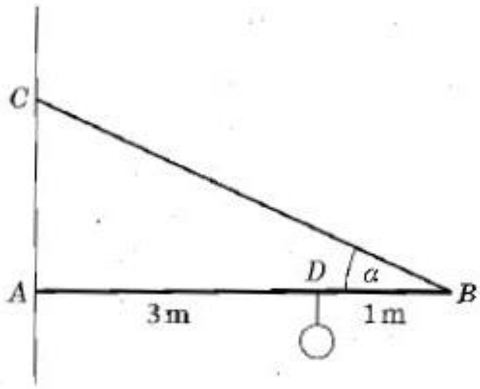
   b Given that the rod is about to slip, find μ.

3. The diagram shows the vertical cross section $ABCD$ through the centre of mass of a uniform rectangular box. The box is resting on a rough horizontal floor and leaning against a smooth vertical wall. The box has mass 25 kg. $AB = 0.5$ m, $BC = 1.5$ m and AD is at an angle of θ to the horizontal. The coefficient of friction between the box and the ground is $\frac{1}{4}$. Given that the box is about to slip, find the value of θ.



4. A uniform ladder AB of mass M kg and length 5 m rests with end A on a smooth horizontal floor and end B against a smooth vertical wall. The ladder is held in equilibrium at an angle θ to the floor by a light horizontal string attached to the wall and to a point C on the ladder. If $\tan\theta = 2$, find the tension in the string when the length AC is 2 m.

5. A uniform steel girder AB of weight 400 N and length 4 m, is freely hinged at A to a vertical wall. The girder is supported in a horizontal position by a steel cable attached to the girder at B. The other end of the cable is attached to the point C vertically above A on the wall, with $\angle ABC = \alpha$ where $\tan\alpha = \frac{1}{2}$. A load of weight 200 N is suspended by another cable from the girder at the point D, where $AD = 3$ m, as shown in the diagram. The girder remains horizontal and in equilibrium. The girder is modelled as a rod, and the cables as light inextensible strings.



a Show that the tension in the cable BC is $350\sqrt{5}$ N.

b Find the magnitude of the reaction on the girder at A.

6. A smooth sphere S of mass m is moving with speed u on a smooth horizontal plane. The sphere S collides with another smooth sphere T, of equal radius to S but of mass km, moving in the same straight line and in the same direction with speed $\lambda u, 0 < \lambda < \frac{1}{2}$. The coefficient of restitution between S and T is e.

Given that S is brought to rest by the impact,

a) show that $e = \dfrac{1+k\lambda}{k(1-\lambda)}$.

b) Deduce that $k > 1$.

7. A particle A, of mass $2m$, is moving with speed u on a horizontal table when it collides directly with a particle B, of mass $3m$, which is at rest. The coefficient of restitution between the particles is e.

a) Find, in terms of e and u, the velocities of A and B immediately after the collision.

b) Show that, for all possible values of e, the speed of A immediately after the collusion is not greater than $\frac{2}{5}u$.

Given that the magnitude of the impulse exerted by B on A is $\frac{11}{5}mu$,

c) find the value of e.

8. A smooth sphere A of mass m is moving with speed u on a smooth horizontal table when it collides directly with another smooth sphere B of mass $3m$, which is at rest on the table. The coefficient of restitution between A and B is e. The spheres have the same radius and are modelled as particles.

a) Show that the speed of B immediately after the collision is $\frac{1}{4}(1+e)u$.

b) Find the speed of A immediately after the collision.

Immediately after the collision the total kinetic energy of the spheres is $\frac{1}{6}mu^2$.

c) Find the value of e.

d) Hence show that A is at rest after the collision.

9. A particle P of mass $3m$ is moving with speed $2u$ in a straight line on a smooth horizontal table. The particle P collides with a particle Q of mass $2m$ moving with speed u in the opposite direction to P. the coefficient of restitution between P and Q is e.

a) Show that the speed of Q after the collision is $\frac{1}{5}u(9e+4)$

As a result of the collision, the direction of motion of P is reversed.

b) Find the range of possible values of e.

Given that the magnitude of the impulse of P on Q is $\frac{32}{5}mu$,

c) find the value of e.

CONSOLIDATION – FP2

10. $$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+4\frac{\mathrm{d}y}{\mathrm{d}x}+5y=65\sin 2x,\ x>0$$

a) Find the general solution of the differential equation.

b) Show that for large values of x this general solution may be approximated by a sine function and find this sine function.

11. a) Find the general solution of the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x}+2y=x$

Given that $y = 1$ at $x = 0$,

b) find the exact values of the coordinates of the minimum point of the particular solution curve,

c) draw a sketch of this particular solution curve.

12. a) Sketch the graph of $y = |x - 2a|$, given that $a > 0$.

b) Solve $|x - 2a| > 2x + a$, where $a > 0$.

13. a) Express $\dfrac{1}{r(r+2)}$ in partial fractions.

b) Hence prove, by the method of differences, that $\displaystyle\sum_{r=1}^{n} \frac{4}{r(r+2)} = \frac{n(3n+5)}{(n+1)(n+2)}$.

c) Find the value of $\displaystyle\sum_{r=50}^{100} \frac{4}{r(r+2)}$, to 4 decimal places.

14. The curve C has polar equation $\quad r = 6\cos\theta, \quad -\dfrac{\pi}{2} \le \theta < \dfrac{\pi}{2}$,

and the line D has polar equation $\quad r = 3\sec\left(\dfrac{\pi}{3} - \theta\right), \quad -\dfrac{\pi}{6} \le \theta \dfrac{5\pi}{6}$.

a) Find a Cartesian equation of C and a Cartesian equation of D.

b) Sketch on the same diagram the graphs of C and D, indicating where each cuts the initial line.

The graphs of C and D intersect at the points P and Q.

c) Find the polar coordinates of P and Q.

15. In the Argand diagram the point P represents the complex number z.

Given that $\arg\left(\dfrac{z - 2i}{z + 2}\right) = \dfrac{\pi}{2}$,

a) sketch the locus of P,

b) deduce the value of $|z + 1 - \mathrm{i}|$.

The transformation T from the z-plane to the w-plane is defined by $w = \dfrac{2(1+\mathrm{i})}{z+2}, \quad z \ne -2$.

c) Show that the locus of P in the z-plane is mapped to part of a straight line in the w-plane, and show this in an Argand diagram.

CHALLENGE QUESTION

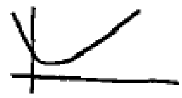
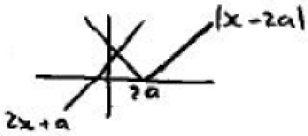
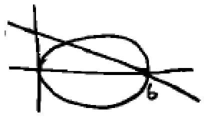
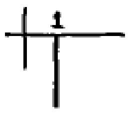
Prove that the rectangle of greatest perimeter which can be inscribed in a given circle is a square.

The result changes if, instead of maximising the sum of lengths of sides of the rectangle, we seek to maximise the sum of nth powers of the lengths of those sides for $n \geqslant 2$. What happens if $n = 2$? What happens if $n = 3$? Justify your answers.

Answers:
Current work:

1a)	41.6°	1b)	24.0°	2a)	$\frac{15\sqrt{3}}{4}$ N, $\frac{25\sqrt{3}}{4}$ N
2b)	$\frac{1}{\sqrt{3}}$	3)	50.2°	4)	$\frac{5Mg}{12}$
5)	743 N	7a)	$v_B = \frac{2u}{5}(1+e)$ $v_A = \frac{u}{5}(2-3e)$	7c)	$e = \frac{5}{6}$
8b)	$\frac{1}{4}(1-3e)u$	8c)	$e = \frac{1}{3}$	9b)	$\frac{2}{3} < e \leq 1$

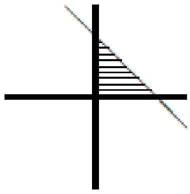
Consolidation

10a)	$y = e^{-2x}(A\cos x + B\sin x)$ $+$ [illegible] $\sin 2x - 8\cos 2x$	10b)	$y \to \sqrt{65}\sin(2x - 83^\circ)$	11a)	$y = \frac{1}{2}x - \frac{1}{4} + ce^{-2x}$
11b)	$\left(\frac{1}{2}\ln 5, \frac{1}{4}\ln 5\right)$	11c)		12a)	
12b)	$x < \frac{1}{3}a$	13a)	$\frac{1}{2r} - \frac{1}{2(r+2)}$	13c)	0.0398 (4 dp)
14a)	$\frac{x}{2} + \frac{\sqrt{3}}{2}y = 3$	14b)		14c)	$(6,0)$ $\left(3, \frac{\pi}{3}\right)$
15b)	$\sqrt{2}$	15c)			

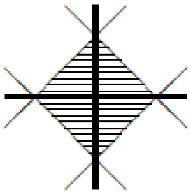
Challenge question: sigma

The way to deal with the modulus signs in this question is to consider first the case when the things inside the modulus signs are positive, and then get the full picture by symmetry, shifting the origin as appropriate.

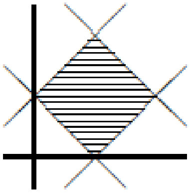
For part (i), consider the the first quadrant $x \geqslant 0$ and $y \geqslant 0$. In this quadrant, the inequality is $x + y \leqslant 1$. Draw the line $x + y = 1$ and then decide which side of the line is described by the inequality. It is obviously (since x has to be smaller than something) the region to the left of the line; or (since y also has to be smaller than something) the region below the line, which is the same region.



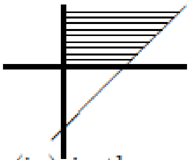
Similar arguments could be used in the other quadrants, but it is easier to note that the inequality $|x| + |y| \leqslant 1$ unchanged when x is replaced by $-x$, or y is replaced by $-y$, so the sketch should have reflection symmetry in both axes.

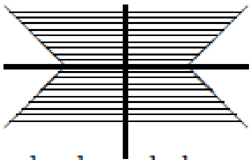


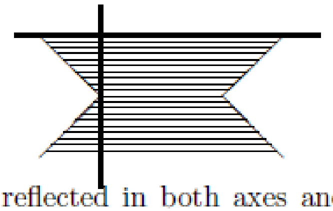
Part (ii) is the same as part (i), except that the origin is translated to $(1, 1)$.



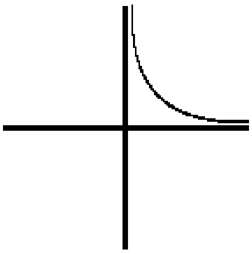
For part (iii), consider first $x - y \leqslant 1$, where $x > 0$ and $y > 0$, which give an area that is infinite in extent in the positive y direction. Then reflect this in both axes and translate one unit down the y axis and 1 unit along the positive x axis.

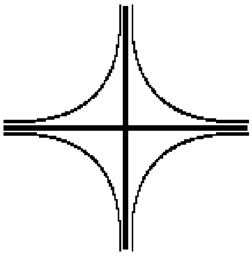


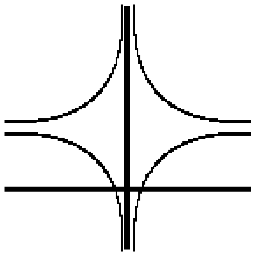




Part (iv) is the rectangular hyperbola $xy = 1$ reflected in both axes and the region required is enclosed by the four hyperbolas. The four hyperbolas are then translated 2 units up the y axis.







BHASVIC

MαTHS

ASSIGNMENT COVER SHEET sigma

Name ________________________________ Maths Teacher ________________________________

Question	Done	Backpack	Ready for test	Notes
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