α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	**o**	π	ρ	σ	τ	υ	φ	χ	ψ	ω

"The mathematician's patterns, like the painter's or the poet's, must be beautiful: the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test." G H Hardy

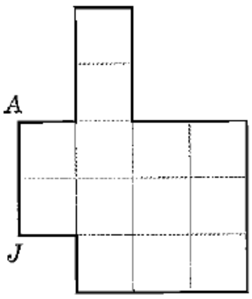
Further Maths A2 (M2FP2D1) Assignment o (omicron) A due in w/b 15th January 18

PREPARATION *Every week you will be required to do some preparation for future lessons, to be advised by your teacher.*

CURRENT WORK – MECHANICS Centres of Mass

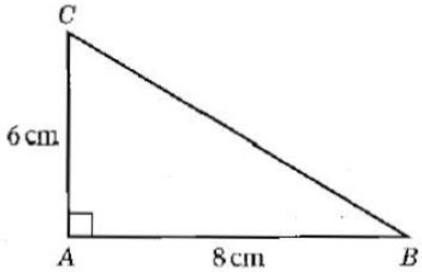
Suspension and Tilting

1. The centre of mass of four particles of masses $2m$, $3m$, $7m$ and $8m$, which are positioned at the points $(0, a)$, $(0, 2)$, $(0, -1)$ and $(0, 1)$ respectively, is the point G. Given that the coordinates of G are $(0, 1)$, find the value of a.

2. 

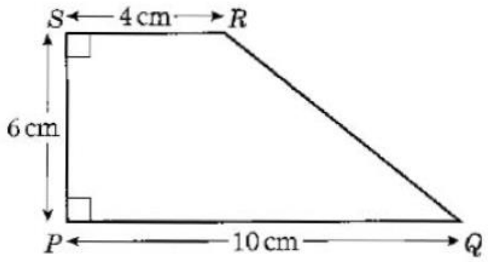
The lamina is freely suspended from the point A and hangs in equilibrium.
Find the angle between AJ and the downward vertical.

3. The uniform triangular lamina ABC shown below is placed on a rough plane inclined at an angle α to the horizontal.



The edge AB is in contact with the plane, with A below B.
Given that the lamina is on the point of toppling about A, find the value of α.

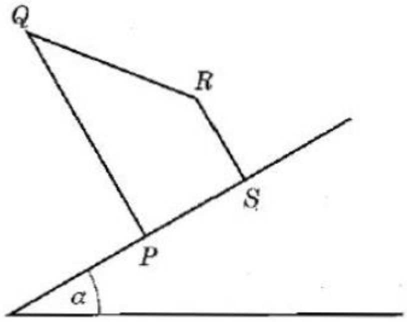
4. *PQRS* is a uniform lamina.



a Find the distance of the centre of mass of the lamina from

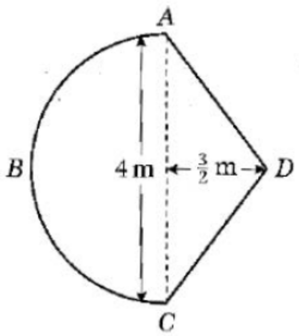
i *PS* **ii** *PQ*.

b The diagram shows the lamina on a rough inclined plane of angle α.



Given that the lamina is about to topple about the point *P*, find the value of α, giving your answer to 3 s.f.

5. The diagram shows a uniform lamina consisting of a semi-circle joined to a triangle *ADC*.



The sides *AD* and *DC* are equal.

a Find the distance of the centre of mass of the lamina from *AC*.

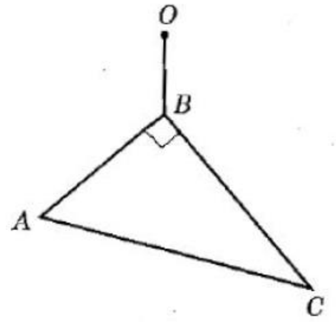
The lamina is freely suspended from *A* and hangs at rest.

b Find, to the nearest degree, the angle between *AC* and the vertical.

The mass of the lamina is *M*. A particle *P* of mass *kM* is attached to the lamina at *D*. When suspended from *A*, the lamina now hangs with its axis of symmetry, *BD*, horizontal.

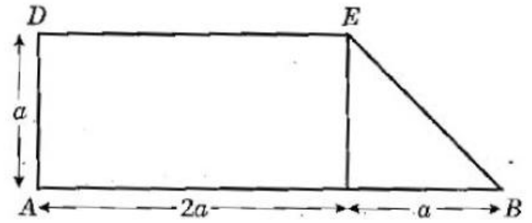
c Find, to 3 s.f., the value of *k*.

6. A uniform triangular lamina ABC is in equilibrium, suspended from a fixed point O by a light inextensible string attached to the point B of the lamina, as shown in the diagram.

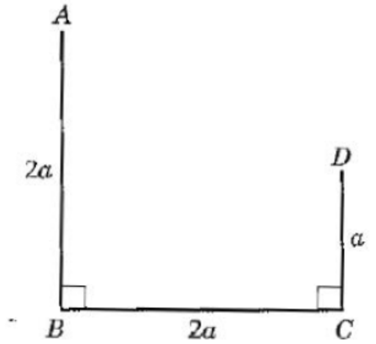


Given that $AB = 9$ cm, $BC = 12$ cm and $A\hat{B}C = 90°$, find the angle between BC and the downward vertical.

7. A uniform rectangular piece of card $ABCD$ has $AB = 3a$ and $BC = a$. One corner of the rectangle is folded over to form a trapezium $ABED$ as shown in the diagram:



8. A thin uniform wire of length $5a$ is bent to form the shape $ABCD$, where $AB = 2a$, $BC = 2a$, $CD = a$ and BC is perpendicular to both AB and CD, as shown in the diagram:



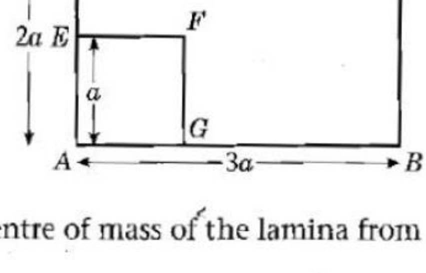
a Find the distance of the centre of mass of the wire from

i AB, **ii** BC.

The wire is freely suspended from B and hangs at rest.

b Find, to the nearest degree, the angle between AB and the vertical.

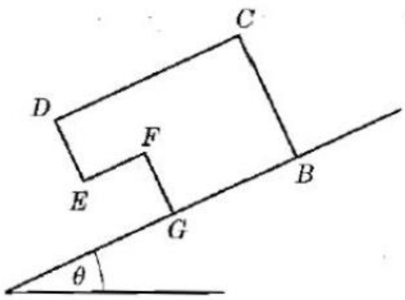
9. A uniform lamina consists of a rectangle $ABCD$, where $AB = 3a$ and $AD = 2a$, with a square hole $EFGA$, where $EF = a$, as shown in the diagram:



a Find the distance of the centre of mass of the lamina from

i AD, **ii** AB.

The lamina is balanced on a rough plane inclined to the horizontal at an angle θ. The plane of the lamina is vertical and the inclined plane is sufficiently rough to prevent the lamina from slipping. The side GB is in contact with the plane with G lower than B, as shown in the diagram:



b Find, in degrees to 1 decimal place, the greatest value of θ for which the lamina can rest in equilibrium without toppling.

CONSOLIDATION – FP2

10. $$y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+y=0$$

a) Find an expression for $\frac{d^3y}{dx^3}$.

Given that $y = 1$ and $\frac{dy}{dx}=1$ at $x = 0$,

b) find the series solution for y, in ascending powers of x, up to and including the term in x^3.

c) Comment on whether it would be sensible to use your series solution to give estimates for y at $x = 0.2$ and at $x = 50$.

11. a) Use the substitution $y = vx$ to transform the equation $\frac{dy}{dx}=\frac{(4x+y)(x+y)}{x^2}, x>0$ (I) into the equation

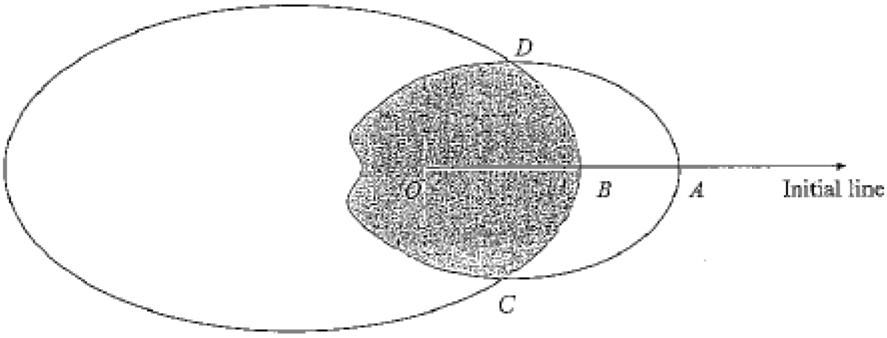
$$x\frac{dv}{dx}=(2+v)^2 \text{ (II)}.$$

b) Solve the differential equation II to find v as a function of x.

c) Hence show that $y=-2x-\frac{x}{\ln x+c}$, where c is an arbitrary constant, is a general solution of the differential equation I.

12. Solve the inequality $\frac{1}{2x+1}>\frac{x}{3x-2}$.

13.



A logo is designed which consists of two overlapping closed curves.

The polar equations of these curves are

$r = a(3+2\cos\theta)$ and

$r = a(5-2\cos\theta), 0 \le \theta < 2\pi$

Above is a sketch (not to scale) of these two curves.

a) Write down the polar coordinates of the points A and B where the curves meet the initial line.

b) Find the polar coordinates of the points C and D where the two curves meet.

c) Show that the area of the overlapping region, which is shaded in the figure is $\frac{a^2}{3}(49\pi - 48\sqrt{3})$

14. Prove by the method of differences that $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1), n > 1$.

CHALLENGE QUESTION

Show that, if $\tan^2\theta = 2\tan\theta + 1$, then $\tan 2\theta = -1$.

Find all solutions of the equation

$$\tan\theta = 2 + \tan 3\theta$$

which satisfy $0 < \theta < 2\pi$, expressing your answers as rational multiples of π.

Find all solutions of the equation

$$\cot\phi = 2 + \cot 3\phi$$

which satisfy $-\frac{3\pi}{2} < \phi < \frac{\pi}{2}$, expressing your answers as rational multiples of π.

Optional work
Need more M2 practice?

Try the exam paper below.

1. A particle is projected with speed u at an angle of α above the horizontal, and moves freely under gravity alone.

(a) Show that the particle returns to the height at which it was launched after it has travelled a horizontal distance of

$$\frac{u^2 \sin 2\alpha}{g}.$$

On a shooting range, the firing point and the target are at the same height. The range of a rifle is adjusted by altering the angle to the horizontal at which the bullet is projected. The bullet may be modelled as a particle moving freely under gravity with an initial speed of $400\,\text{ms}^{-1}$.

(b) Find the angle, in degrees to 3 significant figures, at which the bullet should be fired to give the rifle a range of 500 m.

(c) A shot is aimed correctly at a target which is 100 m away, but the sights have been left set at a range of 500 m. Find the height above the point of aim at which the target is hit.

2. An athlete is running a 100-m race. The athlete starts from rest and the race is complete when the athlete has covered a distance of 100 m. In a first model, the athlete is assumed to have a constant acceleration of $2\,\text{ms}^{-2}$ for the first 6 s of the race and to move at constant velocity after that.

(a) Sketch a velocity–time graph and find the time taken for the athlete to finish the race using this model.

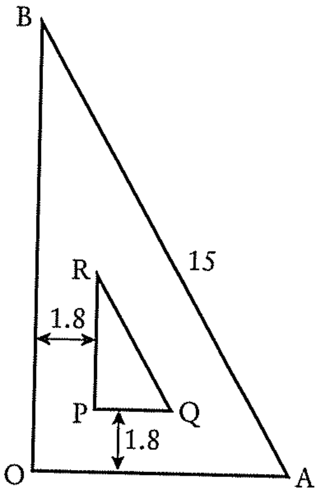
In a second model, the athlete is assumed to have acceleration $a\,\text{ms}^{-2}$ at time t s for the entire duration of the race, where $a = 3.5 - 0.5t$.

(b) Find expressions for the velocity and position at time t using the second model.

(c) Find the position of the athlete at time $t = 11$ using the second model and comment on your result.

(d) Find the times between which the athlete has a velocity greater than $12\,\text{ms}^{-1}$ using the second model.

3. A set square is made from uniform plastic, and is in the shape of a right-angled triangle OAB, with a right angle at O, and angles of 60° at A and 30° at B. The hypotenuse AB is of length 15 cm.
A triangle PQR is cut from the centre. PQ is of length 2.6 cm, and is parallel to OA and 1.8 cm away from OA. PR is of length 4.5 cm and is parallel to OB and 1.8 cm away from OB.
The set square is illustrated in the diagram.



Find the distance of the centre of mass

(a) from OB

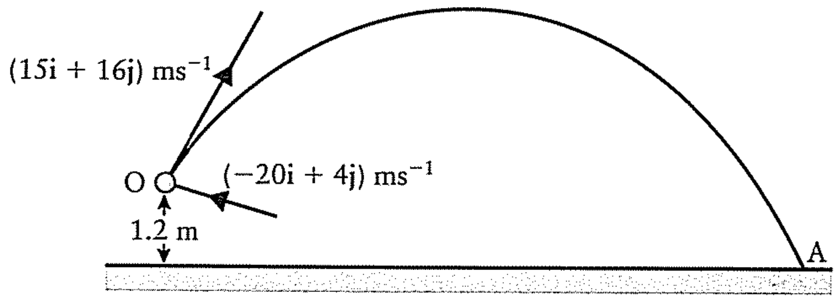
(b) from OA.

The set square is hung on a smooth peg at R.

(c) Find, in degrees to 1 decimal place, the acute angle which OB makes with the vertical.

4.

5.



A ball B of mass 0.4 kg is struck by a bat at a point O which is 1.2 m above horizontal ground. The unit vectors **i** and **j** are respectively horizontal and vertical. Immediately before being struck, B has velocity $(-20\mathbf{i} + 4\mathbf{j})\text{ ms}^{-1}$. Immediately after being struck it has velocity $(15\mathbf{i} + 16\mathbf{j})\text{ ms}^{-1}$.

After B has been struck, it moves freely under gravity and strikes the ground at the point A, as shown in the diagram. The ball is modelled as a particle.

(a) Calculate the magnitude of the impulse exerted by the bat on B.

(b) By using the principle of conservation of energy, or otherwise, find the speed of B when it reaches A.

(c) Calculate the angle which the velocity of B makes with the ground when B reaches A.

(d) State two additional physical factors which could be taken into account in a refinement of the model of the situation which would make it more realistic.

Answers

Current:

1)	$6\frac{1}{2}$	2)	63.0° (3 s.f.)	3)	α = 53°
4a)	**i)** $\frac{26}{7}$ cm **ii)** $\frac{18}{7}$ cm	4b)	α = 34.7°	5a)	0.413m (3 s.f.)
5b)	13° (nearest degree)	5c)	0.275 (3 s.f.)	6)	$\theta = 36.9°$
7a)	$\frac{13a}{9}$	7b)	$\frac{4a}{9}$	7c)	45°
7d)	$m = \frac{5M}{9}$	8a)	**i)** $\frac{4a}{5}$ **ii)** $\frac{a}{2}$	8b)	θ =58°
9a)	**i)** 1.7a **ii)** 1.1a	9b)	θ =32.5°		

Consolidation:

10a)	$\frac{d^3y}{dx^3} = \frac{1}{y}\left[-3\frac{dy}{dx}\frac{d^2y}{dx^2} - \frac{dy}{dx}\right]$	10b)	$y = 1 + x - x^2 + \frac{5}{6}x^3 \ldots$	10c)	must be close to zero
11b)	$v = \frac{-1}{\ln x + c} - 2$	12)	$-\frac{1}{2} < x < \frac{2}{3}$	13a)	$(5a, 0)$, $(3a, 0)$
13b)	$(4a, \frac{\pi}{3})$, $(4a, \frac{5\pi}{3})$				

Challenge Question: lambda

First we put the equations into a more manageable form. Each equality can be written in the form

$$3 = x_n + \frac{2}{x_{n+1}}, \qquad \text{i.e.} \qquad x_{n+1} = \frac{2}{3 - x_n}\ .$$

We find $x_1 = 2/3$, $x_2 = 6/7$, $x_3 = 14/15$ and $x_4 = 30/31$. The denominators give the game away. We guess

$$x_n = \frac{2^{n+1} - 2}{2^{n+1} - 1}\ .$$

For the induction, we need a starting point: our guess certainly holds for $n = 1$ (and 2, 3, and 4!).

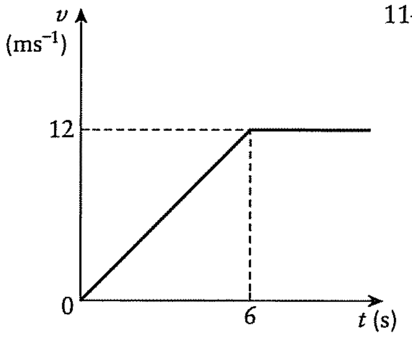
For the inductive step, we suppose our guess also holds for $n = k$, where k is any integer. If we can show that it then also holds for $n = k + 1$, we are done.

We have, from the equation given in the question,

$$x_{k+1} = \frac{2}{3 - x_k} = \frac{2}{3 - \frac{2^{k+1}-2}{2^{k+1}-1}} = \frac{2(2^{k+1} - 1)}{3(2^{k+1} - 1) - (2^{k+1} - 2)} = \frac{2^{k+2} - 2}{2^{k+2} - 1}\ ,$$

Optional work

1. **(b)** 0.877°
   (c) 1.23 m

2. (a) 

$11\frac{1}{3}$ s

(b) $v = 3.5t - 0.25t^2$, $x = \dfrac{7t^2}{4} - \dfrac{t^3}{12}$

(c) 100.8 m. The race has finished.

(d) $6 < t < 8$

3.

(a) 2.48 cm

(b) 4.52 cm

(c) 20.8°

4.

(a) 294 W

(b) $P = 294 + \frac{5}{6}R$

(c) $P = 588$, $R = 353$

5.

(a) 14.8 Ns

(b) 22.5 ms^{-1}

(c) 48.1°

(d) Air resistance, spin

6.

12 (b) 877 N

(c) By making the tension at D equal to the weight of the load.

BHASVIC

MATHS

ASSIGNMENT COVER SHEET omicron

Name ________________________________ Maths Teacher ____________________________

Question	Done	Backpack	Ready for test	Notes
1				
2				
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