α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	$\boxed{\xi}$	o	π	ρ	σ	τ	υ	φ	χ	ψ	ω

"The mathematician's patterns, like the painter's or the poet's, must be beautiful: the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test."

G H Hardy

Further Maths A2 (M2FP2D1) Assignment ξ (xi) A

Due in w/b 8th January

PREPARATION *Every week you will be required to do some preparation for future lessons, to be advised by your teacher.*

CURRENT WORK – MECHANICS

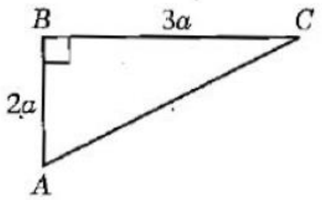
1. A light rectangular piece of card $ABCD$ has $AB = 8$ cm and $AD = 6$ cm. Three particles of mass 3 g, 2 g and 2 g are attached to the rectangle at the points A, B and C respectively.
   a Find the mass of a particle which must be placed at the point D for the centre of mass of the whole system of four particles to lie 3 cm from the line AB.
   b With this fourth particle in place, find the distance of the centre of mass of the system from the side AD.

2. Find the position of the centre of mass of a uniform semi-circular lamina of radius 4 cm and centre O.

3. The centre of mass of a uniform triangular lamina ABC is at the point $(2, a)$. Given that A is the point $(4, 3)$, B is the point $(b, 1)$ and C is the point $(-1, 5)$, find the values of a and b.

4. Find the position of the centre of mass of the following uniform triangular laminas:

a



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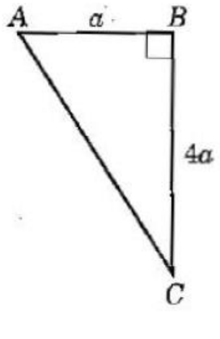

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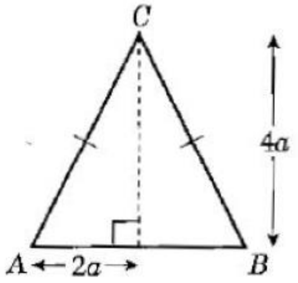
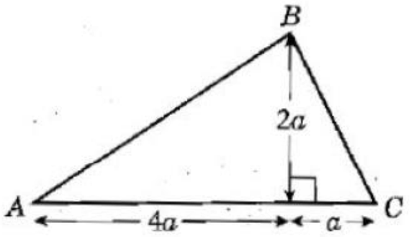
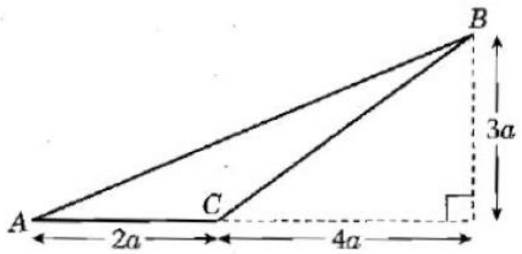


d



e



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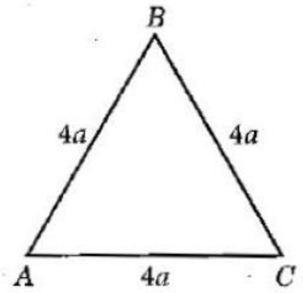
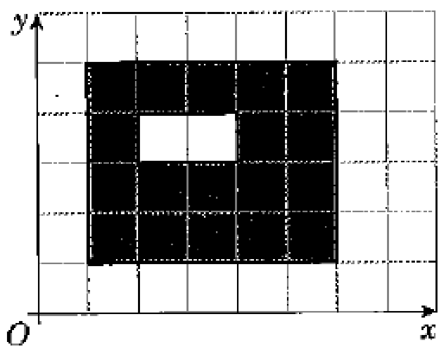


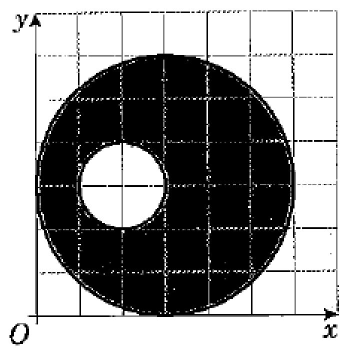
5. A uniform triangular lamina is isosceles and has the line $y = 4$ as its axis of symmetry. One of the vertices of the triangle is the point (2, 1). Given that the x-coordinate of the centre of mass of the lamina is -3, find the coordinates of the other two vertices.

6. The following diagrams show uniform plane figures. Each one is drawn on a grid of unit squares. Find, in each case, the coordinates of the centre of mass.

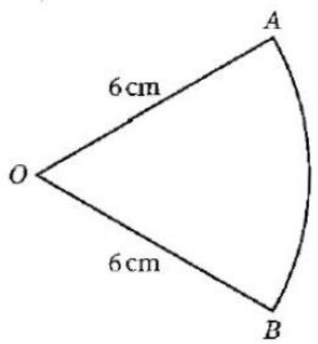
a)



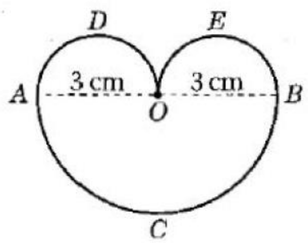
b)



7. Find the position of the centre of mass of the framework shown in the diagram which is formed by bending a uniform piece of wire of total length $(12 + 2\pi)$ cm to form a sector of a circle, centre O, radius 6 cm.



8. A uniform length of wire is bent to form the shape shown in the diagram:



ACB is a semicircle of radius 3 cm, centre O.
ADO and BEO are both semicircles of radius 1.5 cm.
Find the position of the centre of mass of the framework.

9. Four particles P, Q, R and S of masses 3 kg, 5 kg, 2 kg and 4 kg are placed at the points (1, 6), (−1, 5), (2, −3) and (−1, −4) respectively. Find the coordinates of the centre of mass of the particles.

CONSOLIDATION – FP2

10. a) Find the general solution of the differential equation $\cos x \frac{\mathrm{d}y}{\mathrm{d}x} + (\sin x)\, y = \cos^3 x$.

b) Show that, for $0 \le x \le 2\pi$, there are two points on the x-axis through which all the solution curves for this differential equation pass.

c) Sketch the graph, for $0 \le x \le 2\pi$, of the particular solution for which y = 0 at x = 0.

11. a) Find the general solution of the differential equation $2\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + 7\frac{\mathrm{d}y}{\mathrm{d}t} + 3y = 3t^2 + 11t$

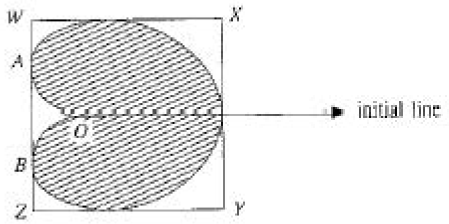
b) Find the particular solution of this differential equation for which y = 1 and $\frac{\mathrm{d}y}{\mathrm{d}t} = 1$ when t = 0.

c) For this particular solution, calculate the value of y when t = 1.

12. a) Sketch, on the same axes, the graphs with equation $y = |2x - 3|$, and the line with equation $y = 5x - 1$.

b) Solve the inequality $|2x - 3| < 5x - 1$.

13.



The figure above shows a sketch of the cardioid C with equation $r = a(1 + \cos\theta), -\pi < \theta \le \pi$. Also shown are the tangents to C that are parallel and perpendicular to the initial line. These tangents form a rectangle $WXYZ$.

a) Find the area of the finite region, shaded in the figure above, bounded by the curve C.

b) Find the polar coordinates of the points A and B where WZ touches the curve C.

c) Hence find the length of WX.

Given that the length of WZ is $\frac{3\sqrt{3}a}{2}$,

d) find the area of the rectangle $WXYZ$.

A heart-shape is modelled by the cardioid C, where a = 10 cm. The heart shape is cut from the rectangular card $WXYZ$, shown in the figure above.

e) Find a numerical value for the area of card wasted in making this heart shape.

14. (i) a) On the same Argand diagram sketch the loci given by the following equations.

$$|z-1|=1,$$

$$\arg(z+1)=\frac{\pi}{12},$$

$$\arg(z+1)=\frac{\pi}{2}$$

b) Shade on your diagram the region for which $|z-1|\leq$ and $\frac{\pi}{12}\leq\arg(z+1)\leq\frac{\pi}{2}$.

(ii) a) Show that the transformation $w=\frac{z-1}{z}$, $z\neq 0$, maps $|z-1|=1$ in the z-plane onto $|w|=|w-1|$ in the w-plane.

The region $|z-1|\leq 1$ in the z-plane is mapped onto the region T in the w-plane.

b) Shade the region T on an Argand diagram.

CHALLENGE QUESTION

Suppose that

$$3=\frac{2}{x_1}=x_1+\frac{2}{x_2}=x_2+\frac{2}{x_3}=x_3+\frac{2}{x_4}=\cdots.$$

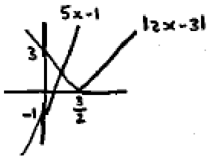
Guess an expression, in terms of n, for x_n. Then, by induction or otherwise, prove the correctness of your guess.

Answers:

Current:

1a)	3 g	1b)	3.2 cm
2)	Centre of mass is on the axis of symmetry at a distance $\frac{16}{3\pi}$ from the centre	3)	$a = 3 \quad b = 3$
4a)	Distance a from AB Distance $\frac{2a}{3}$ from BC	4b)	Distance $\frac{a}{3}$ from BC Distance $\frac{4a}{3}$ from AB
4c)	On line of symmetry, $\frac{4a}{3}$ from the line AB	4d)	$\left(3a, \frac{2a}{3}\right)$ with A as origin and AC as x-axis
4e)	$\left(\frac{8a}{3}, a\right)$ with A as origin and AC as x-axis	4f)	$\left(2a, \frac{2a\sqrt{3}}{3}\right)$ with A as origin and AC as x-axis
5)	(2, 7) and (-13, 4)	6a)	$\left(\frac{32}{9}, \frac{53}{18}\right)$
6b)	$\left(\frac{25}{8}, 3\right)$	7)	Centre of mass is on line of symmetry through O, and a distance of $\frac{9\left(\sqrt{3}+2\right)}{6+\pi}$ from O
8)	Centre of mass is on the line of symmetry at a distance of $\frac{3}{2\pi}$ below the line AB	9)	$\left(-\frac{1}{7}, \frac{3}{2}\right)$

Consolidation:

10)	$y\sec x = \sin x + c$	11a)	$y = Ae^{-\frac{1}{2}t} + Be^{-3t} + (t^2 - t + 1)$	11b)	$y = \frac{4}{5}\left(e^{-\frac{1}{2}t} - e^{-3t}\right) + (t^2 - t + 1)$
11c)	$y = \frac{4}{5}\left(e^{-\frac{1}{2}} - e^{-3}\right) + 1$ $= 1.445$ (3 dp)	12a)		12b)	$x > \frac{4}{7}$
13a)	$\frac{3\pi}{2}a^2$	13b)	$\left(\frac{9}{2}, -\frac{2\pi}{3}\right)$	13c)	$\frac{9}{4}a$
13d)	$\frac{27\sqrt{3}}{8}a^2$	13e)	113.3 cm^2		

Challenge Question: kappa

Start by labelling the equations:

$$2yz + zx - 5xy = 2\,, \qquad (1)$$
$$yz - zx + 2xy = 1\,, \qquad (2)$$
$$yz - 2zx + 6xy = 3\,. \qquad (3)$$

We use Gaussian elimination. Rearranging equation (1) gives

$$yz = -\tfrac{1}{2}zx + \tfrac{5}{2}xy + 1\,, \qquad (4)$$

which we substitute back into equations (2) and (3) :

$$-\tfrac{3}{2}zx + \tfrac{9}{2}xy = 0\,, \qquad (5)$$
$$-\tfrac{5}{2}zx + \tfrac{17}{2}xy = 2\,. \qquad (6)$$

Thus $zx = 3xy$ (using equation (5)). Substituting into equation (6) gives $xy = 2$ and $zx = 6$. Finally, substituting back into equation (1) shows that $yz = 3$.

The question is now plain sailing. Multiplying the three values together gives $(xyz)^2 = 36$ and taking the square root gives $xyz = \pm 6$ as required.

Now it remains to solve for x, y and z individually. We know that $yz = 3$, so if $xyz = +6$ then $x = +2$, and if $xyz = -6$ then $x = -2$. The solutions are therefore either $x = +2$, $y = 1$, and $z = 3$ or $x = -2$, $y = -1$, and $z = -3$.

BHASVIC

MαTHS

ASSIGNMENT COVER SHEET xi

Name ______________________________ Maths Teacher ______________________________

Question	Done	Backpack	Ready for test	Notes
1				
2				
3				
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