α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	σ	τ	υ	φ	χ	ψ	ω

"The mathematician's patterns, like the painter's or the poet's, must be beautiful: the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test." G H Hardy

Further Maths A2 (M2FP2D1) Assignment ν (nu) A

Due 3rd January 17

PREPARATION *Every week you will be required to do some preparation for future lessons, to be advised by your teacher.*

CURRENT WORK – MECHANICS: Variable Acceleration

1. At time t seconds, a particle P has position vector $\mathbf{r}$ m with respect to a fixed origin O, where
$$\mathbf{r} = (3t - 4)\mathbf{i} + (t^3 - 4t)\mathbf{j}.$$
Find
a the velocity of P when $t = 3$,
b the acceleration of P when $t = 3$.

2. At time t seconds, the force $\mathbf{F}$ newtons acting on a particle P, of mass 0.5 kg , is given by
$$\mathbf{F} = 3t\mathbf{i} + (4t - 5)\mathbf{j}.$$
When $t = 1$, the velocity of P is $12\mathbf{i}$ m s^{-1}. Find
a the velocity of P after t seconds,
b the angle the direction of motion of P makes with $\mathbf{i}$ when $t = 5$, giving your answer to the nearest degree.

3. At time t seconds (where $t \geqslant 0$), a particle P is moving in a plane with acceleration $(2\mathbf{i} - 2t\mathbf{j})$ m s^{-2}. When $t = 0$, the velocity of P is $2\mathbf{j}$ m s^{-1} and the position vector of P is $6\mathbf{i}$ m with respect to a fixed origin P.
a Find the position vector of P at time t seconds.
At time t seconds (where $t \geqslant 0$), a second particle Q is moving in the plane with velocity $((3t^2 - 4)\mathbf{i} - 2t\mathbf{j})$ m s^{-1}. The particles collide when $t = 3$.
b Find the position vector of Q at time $t = 0$.

4. A particle P moves along the x-axis so that, at time t seconds, the displacement of P from O is x metres and the velocity of P is $v\,\text{m s}^{-1}$, where

$$v = 6t + \tfrac{1}{2}t^3.$$

a Find the acceleration of P when $t = 4$.

b Given also that $x = -5$ when $t = 0$, find the distance OP when $t = 4$.

5. A particle P of mass 0.2 kg is moving in a straight line under the action of a single variable force **F** newtons. At time t seconds the displacement, s metres, of P from a fixed point A is given by $s = 3t + 4t^2 - \frac{1}{2}t^3$.

Find the magnitude of **F** when $t = 4$.

6. A ball, attached to the end of an elastic string, is moving in a vertical line. The motion of the ball is modelled as a particle B moving along a vertical axis so that its displacement, x m, from a fixed point O on the line at time t seconds is given by $x = 0.6\cos\left(\frac{\pi t}{3}\right)$. Find

a the distance of B from O when $t = \frac{1}{2}$,

b the smallest positive value of t for which B is instantaneously at rest,

c the magnitude of the acceleration of B when $t = 1$. Give your answer to 3 significant figures.

7. A particle P is projected from a point on a horizontal plane with speed u at an angle of elevation θ.

a Show that the range of the projectile is $\frac{u^2 \sin 2\theta}{g}$.

b Hence find, as θ varies, the maximum range of the projectile.

c Given that the range of the projectile is $\frac{2u^2}{3g}$, find the two possible value of θ. Give your answers to 0.1°.

8. A particle P passes through a point O and moves in a straight line. The displacement, s metres, of P from O, t seconds after passing through O is given by

$$s = -t^3 + 11t^2 - 24t$$

a Find an expression for the velocity, $v\,\text{m s}^{-1}$, of P at time t seconds.

b Calculate the values of t at which P is instantaneously at rest.

c Find the value of t at which the acceleration is zero.

d Sketch a velocity-time graph to illustrate the motion of P in the interval $0 \leqslant t \leqslant 6$, showing on your sketch the coordinates of the points at which the graph crosses the axes.

e Calculate the values of t in the interval $0 \leqslant t \leqslant 6$ between which the speed of P is greater than $16\,\text{m s}^{-1}$.

9. The position vector of a particle P, with respect to a fixed origin O, at time t seconds (where $t \geqslant 0$) is $\left[\left(6t - \frac{1}{2}t^3\right)\mathbf{i} + (3t^2 - 8t)\mathbf{j}\right]$ m. At time t seconds, the velocity of a second particle Q, moving in the same plane as P, is $(-8\mathbf{i} + 3t\mathbf{j})\,\mathrm{m\,s^{-1}}$.

a Find the value of t at the instant when the direction of motion of P is perpendicular to the x-axis.

b Given that P and Q collide when $t = 4$, find the position vector of Q with respect to O when $t = 0$.

CONSOLIDATION – FP2

10. a) Show that $y = \frac{1}{2}x^2\mathrm{e}^x$ is a solution of the differential equation

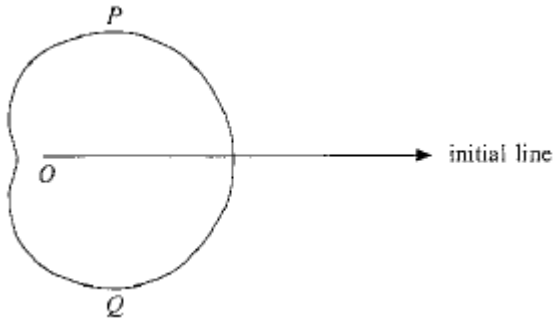
$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} + y = \mathrm{e}^x.$$

b) Solve the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} + y = \mathrm{e}^x$$

Given that at $x = 0$, $y = 1$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 2$.

11. Using algebra, find the set of values of x for which $2x - 5 > \frac{3}{x}$

12. 

The curve C shown above has polar equation $r=a(3+\sqrt{5}\cos\theta)$, $-\pi \leq \theta \leq \pi$

a) Find the polar coordinates of the points P and Q where the tangents to C are parallel to the initial line.

The curve C represents the perimeter of the surface of a swimming pool. The direct distance from P to Q is 20 m.

b) Calculate the value of a.

c) Find the area of the surface of the pool.

13. a) Use de Moivre's theorem to show that $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$.

b) Hence find 3 distinct solutions of the equation $16x^5 - 20x^3 + 5x + 1 = 0$, giving your answers to 3 decimal places where appropriate.

14. Given that $\frac{\mathrm{d}^n}{\mathrm{d}x^n}\left(\mathrm{e}^x \cos x\right) = 2^{\frac{1}{2}n}\mathrm{e}^x \cos\left(x + \frac{1}{4}n\pi\right), n \geq 1$, find the Maclaurin series expansion of $\mathrm{e}^x \cos x$, in ascending powers of x, up to and including the term in x^4.

CHALLENGE QUESTION

Consider the system of equations

$$\begin{aligned} 2yz + zx - 5xy &= 2 \\ yz - zx + 2xy &= 1 \\ yz - 2zx + 6xy &= 3. \end{aligned}$$

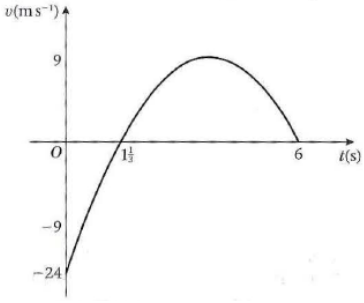
Show that

$$xyz = \pm 6$$

and find the possible values of x, y and z.

Answers:

Current Work

1a)	$(3\mathbf{i} + 23\mathbf{j})$ m s^{-1}	1b)	$18\mathbf{j}$ m s^{-2}	2a)	$\left(\left(3t^2 + 9\right)\mathbf{i} + \left(4t^3 - 10t + 6\right)\mathbf{j}\right)$ m s^{-1}
2b)	34° (nearest degree)	3a)	$\left(\left(t^2 + 6\right)\mathbf{i} + \left(2t - \frac{t^3}{3}\right)\mathbf{j}\right)$ m	3b)	$6\mathbf{j}$ m
4a)	30 m s^{-2}	4b)	OP = 75 m	5	0.8 N
6a)	$0.3\sqrt{3}$ m	6b)	$t = \frac{3}{2}$	6c)	0.329 m s^{-2} (3 s.f.)
7b)	$\frac{u^2}{g}$	7c)	20.9°, 69.1° (nearest 0.1°)	8a)	$\left(-3t^2 + 22t - 24\right)$ m s^{-1}
8b)	$t = \frac{4}{3}, 6$	8c)	$\frac{11}{3}$	8d)	
9a)	t = 2	9b)	$(24\mathbf{i} - 8\mathbf{j})$ m		

Consolidation

10b)	b) $y = e^x(1 + x + \frac{1}{2}x^2)$	11)	$-\frac{1}{2} < x < 0$, $x > 3$	12a)	(4a, 1.107), (4a, -1.107)
12b)	a = 2.795 (3 dp)	12c)	282 m^2 (3 sf)	13b)	$x = -1, -0.309, 0.809$
14)	$1 + x - \frac{1}{3}x^3 - \frac{1}{6}x^4$				

Challenge question: iota

Probably the best approach is to start counting with the arrangements which use as many high denomination coins as possible, then work down.

We can make up 10p as follows:

10;

5+5 (one way using two 5p coins);

5+2+2+1, 5+2+1+1+1, 5+1+1+1+1+1, (three ways using one 5p coin);

2+2+2+2+2, 2+2+2+2+1+1, etc, (six ways using no 5p coins);

making a total of 11 ways.

We can make up 20p as follows:

20;

10 + any of the 11 arrangements in the first part of the question;

5+5+5+5;

5+5+5+2+2+1, etc (3 ways using three 5p coins);

5+5+2+2+2+2+2, 5+5+2+2+2+2+1+1 etc (6 ways using two 5p coins);

5+2+2+2+2+2+2+2+1, etc (8 ways using one 5 and making 15 out of 2p and 1p coins);

2+2+2+2+2+2+2+2+2+2, etc (11 ways of making 20p with 2p and 1p coins).

Grand total = 41.

BHASVIC MATHS

ASSIGNMENT COVER SHEET nu

Name _____ Maths Teacher _____

Question	Done	Backpack	Ready for test	Notes
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