α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	**μ**	ν	ξ	o	π	ρ	σ	τ	υ	φ	χ	ψ	ω

"The mathematician's patterns, like the painter's or the poet's, must be beautiful: the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test." G H Hardy

Further Maths A2 (M2FP2D1) Assignment μ (mu) A

Due in 11th December

****Do your corrections to the FP2 Mock Exam as part of the assignment this week****

PREPARATION

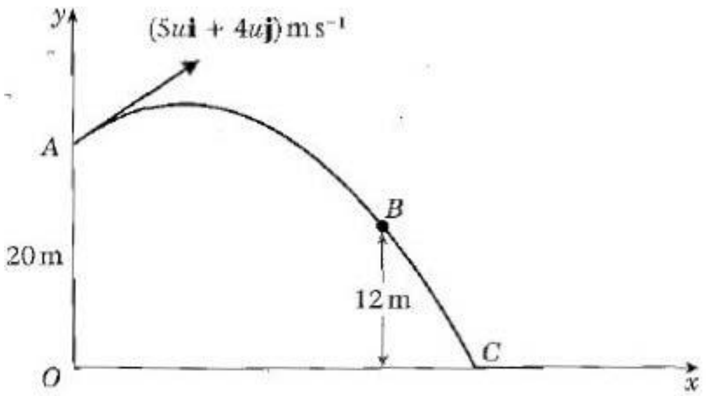
Every week you will be required to do some preparation for future lessons, to be advised by your teacher.

CURRENT WORK - MECHANICS

1. A stone is thrown with speed 30 m s^{-1} from a window which is 20 m above horizontal ground. The stone hits the ground 3.5 s later. Find

   a) the angle of projection of the stone,

   b) the horizontal distance from the window to the point where the stone hits the ground.

2. 

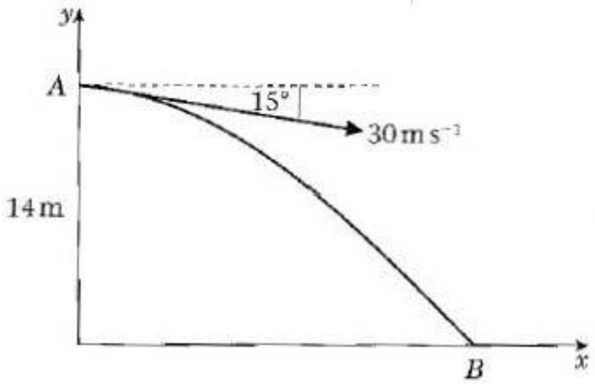
   [In this question, the unit vectors **i** and **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertical]*
   A particle *P* is projected from a point *A* with position vector 20 m with respect to a fixed origin *O*. The velocity of projection is $(5u\mathbf{i} + 4u\mathbf{j})$ m s^{-1}. The particle moves freely under gravity, passing through a point *B*, which has position vector $(k\mathbf{i} + 12\mathbf{j})$ m, where *k* is a constant, before reaching the point *C* on the *x*-axis, as shown in the figure above. The particle takes 4 s to move from *A* to *B*. Find

   a) the value of *u*,

   b) the value of *k*,

   c) the angle the velocity of *P* makes with the *x*-axis as it reaches *C*.

3.



A stone is thrown from a point *A* with speed 30 m s^{-1} at an angle of 15° below the horizontal. The point *A* is 14m above horizontal ground. The stone strikes the ground at the point *B*, as shown in the figure above. Find

a) the time the stone takes to travel from *A* to *B*.

b) the distance *AB*

4. A particle is projected from a point with speed 21 m s^{-1} at an angle of elevation α and moves freely under gravity. When the particle has moved a horizontal distance *x* m, its height above the point of projection is *y* m.

a) Show that $y = x\tan\alpha - \dfrac{x^2}{90\cos^2\alpha}$

b) Given that $y = 8.1$ when *x* = 36, find the value of $\tan\alpha$.

5. A projectile is launched from a point on a horizontal plane with initial speed *u* m s^{-1} at an angle of elevation α. The particle moves freely under gravity until it strikes the plane. The range of the projectile is *R* m.

a) Show that the time of flight of the particle is $\dfrac{2u\sin\alpha}{g}$ seconds.

b) Show that $R = \dfrac{u^2\sin 2\alpha}{g}$.

c) Deduce that, for a fixed *u*, the greatest possible range is when α = 45°.

d) Given that $R = \dfrac{2u^2}{5g}$, find the two possible values of the angle of elevation at which the projectile could have been launched.

6. A particle is projected from a point on level ground with speed *u* m s^{-1} and angle of elevation α. The maximum height reached by the particle is 42 m above the ground and the particle hits the ground 196 m from its point of projection.

Find the value of α and the value of *u*.

7. A ball is thrown from a window above a horizontal lawn. The velocity of projection is 15 m s^{-1} and the angle of elevation is α, where $\tan\alpha = \frac{4}{3}$. The balls takes 4 s to reach the lawn. Find

a) the horizontal distance between the point of projection and the point where the ball hits the lawn,

b) the vertical height above the lawn from which the ball was thrown.

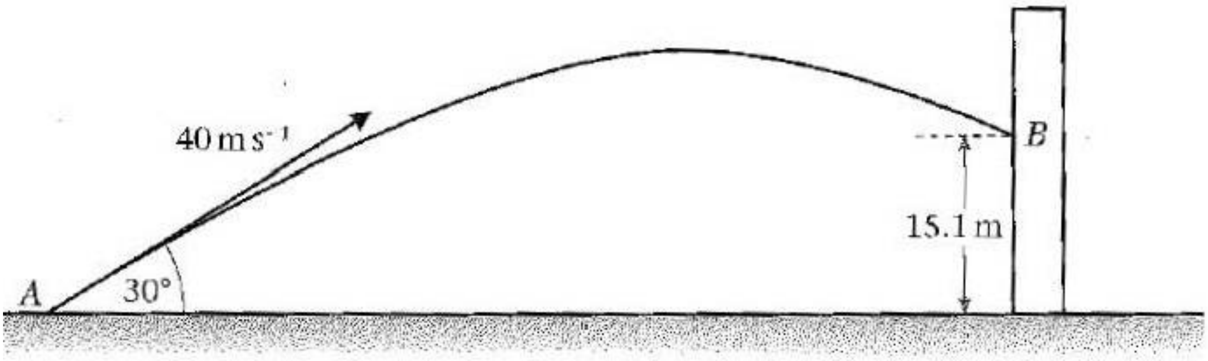
8. A particle P is projected with velocity $(3u\mathbf{i} + 4u\mathbf{j})$ m s^{-1} from a fixed point O on a horizontal plane. Given that P strikes the plane at a point 750 m from O,

a) show that $u = 17.5$,

b) calculate the greatest height above the plane reached by P,

c) find the angle the direction of motion of P makes with $\mathbf{i}$ when $t = 5$.

9.



A golf ball is driven from a point A with a speed of 40 m s^{-1} at an angle of elevation of 30°. On its downward flight, the ball hits an advertising hoarding at a height 15.1 m above the level of A, as shown in the diagram above. Find

a) the time taken by the ball to reach its greatest height above A,

b) the time taken by the ball to travel from A to B,

c) the speed with which the ball hits the hoarding.

CONSOLIDATION – FP2

10. Find the set of values for which $|x-1| > 6x-1$.

11. a) Find the general solution of the differential equation

$$t\frac{\mathrm{d}v}{\mathrm{d}t} - v = t, \quad t > 0$$

and hence show that the solution can be written in the form $v = t(\ln t + c)$, where c is an arbitrary constant.

b) This differential equation is used to model the motion of a particle which has speed v m s^{-1} at time t s. When $t = 2$ the speed of the particle is 3 m s^{-1}. Find, to 3 significant figures, the speed of the particle when $t = 4$.

12. The curve C has polar equation $r = 3a\cos\theta$, $-\frac{\pi}{2} \le \frac{\pi}{2}$. The curve D has polar equation $r = a\left(1+\cos\theta\right), -\pi \le \theta < \pi$. Given that a is a positive constant,

a) sketch, on the same diagram, the graphs of C and D, indicating where each curve cuts the initial line.

The graphs of C intersect at the pole O and at the points P and Q.

b) Find the polar coordinates of P and Q.

c) Use integration to find the exact value of the area enclosed by the curve D and the lines $\theta = 0$ and $\theta = \frac{\pi}{3}$.

The region R contains all points which lie outside D and inside C.

Given that the value of the smaller area enclosed by the curve C and the line $\theta = \frac{\pi}{3}$ is

$\frac{3a^2}{16}\left(2\pi - 3\sqrt{3}\right)$,

d) show that the area of R is πa^2 .

13. a) The point P represents a complex number z in an Argand diagram. Given that

$\left|z - 2\mathrm{i}\right| = 2\left|z + \mathrm{i}\right|$,

(i) find a Cartesian equation for the locus of P, simplifying your answer.

(ii) sketch the locus of P.

b) A transformation T from the z-plane to the w-plane is a translation $-7 + 11\mathrm{i}$ followed by an enlargement with centre the origin and scale factor 3.

Write down the transformation T in the form $w = az + b,\ a, b \in \mathbb{C}$.

14. Prove, by the method of differences, that

$$\sum_{r=2}^{n} \frac{2r-1}{r^2\left(r-1\right)^2} = 1 - \frac{1}{n^2} .$$

CHALLENGE QUESTION

Show that you can make up 10 pence in eleven ways using 10p, 5p, 2p and 1p coins.

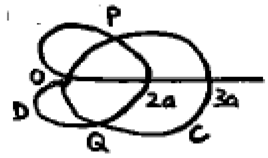
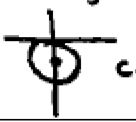
In how many ways can you make up 20 pence using 20p, 10p, 5p, 2p and 1p coins?

Answers:

Current Work

1a)	22° (2 s.f.)	1b)	97 m (2 s.f.
2a)	4.4	2b)	88
2c)	50° (2 s.f.)	3a)	1.1 s (2 s.f.)
3b)	34 m (2 s.f.)	4b)	$\tan\alpha = \frac{5}{4}$
5d)	12° and 78° (nearest degree)	6)	α = 40.6° (nearest 0.1°) u = 44 (2 s.f.)
7a)	36 m	7b)	30 m (2 s.f.)
8a)	250 m	8b)	21.8°
9a)	2.0 s (2 s.f.)	9b)	3.1 s (2 s.f.)
9c)	36 m s^{-1} (2 s.f.)		

Consolidation

10)	$x < \frac{2}{7}$	11a)	$v = t(\ln t + c)$
11b)	8.77	12a)	
12b)	$\left(\frac{3a}{2}, \frac{\pi}{3}\right), \left(\frac{3a}{2}, -\frac{\pi}{3}\right)$	12c)	$\frac{\pi}{4}a^2 + \frac{9\sqrt{3}}{16}a^2$
13a)	$x^2+(y-2)^2 = 4(x^2+(y+1)^2)$  centre(0,-2)	13b)	$w = 3z - 21 + 33i$

Challenge Question: Theta

(i) Writing out the cube explicitly makes it easier to collect up the terms:

$$(1-2x+3x^2-4x^3+5x^4)(1-2x+3x^2-4x^3+5x^4)(1-2x+3x^2-4x^3+5x^4)$$

To get the coefficient of x^6, we need to find all ways of choosing one term from each bracket in such a way that, when multiplied together, the result is a multiple of x^6.

Start by taking the first term (i.e. 1) from the first bracket. This gives

$$1 \times [(3x^2)\times(5x^4)+(-4x^3)\times(-4x^3)+(5x^4)\times(3x^2)]$$

which sums to $46x^6$.

The second term from the first bracket gives a contribution of

$$(-2x)\times[(-2x)\times(5x^4)+(3x^2)\times(-4x^3)+(-4x^3)\times(3x^2)+(5x^4)\times(-2x)] = 88x^6 .$$

The third term from the first bracket gives a contribution of

$$(3x^2)\times[(1)\times(5x^4)+(-2x)\times(-4x^3)+(3x^2)\times(3x^2)+(-4x^3)\times(-2x)+(5x^4)\times(1)] = 105x^6 .$$

The fourth term from the first bracket gives a contribution of

$$(-4x^3)\times[(1)\times(-4x^3)+(-2x)\times(3x^2)+(3x^2)\times(-2x)+(-4x^3)\times(1)] = 80x^6 .$$

The fifth term from the first bracket gives a contribution of

$$(5x^4)\times[(1)\times(3x^2)+(-2x)\times(-2x)+(3x^2)\times(1)] = 50x^6 .$$

The grand total is therefore $369x^6$ and 369 is the required coefficient.

(ii) The given expression is the beginning of the expansion of $\left((1+x)^{-2}\right)^3$, i.e. of $(1+x)^{-6}$. But

$$(1+x)^{-6} = 1-6x+\frac{(-6)\times(-7)}{2!}x^2+\cdots+\frac{11!}{6!\times 5!}x^6+\cdots+(-1)^n\frac{(n+6-1)!}{n!\times 5!}x^n+\ldots$$

so the coefficient of x^6 is $11!/6!5! = 462$.

BHASVIC MαTHS

ASSIGNMENT COVER SHEET mu

Name ______________________________ Maths Teacher ______________________________

Question	Done	Backpack	Ready for test	Notes
1				
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