α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	σ	τ	υ	φ	χ	ψ	ω

"The mathematician's patterns, like the painter's or the poet's, must be beautiful: the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test." G H Hardy

Further Maths A2 (M2FP2D1) Assignment λ (lambda) A

Due in 4th December

PREPARATION - STATICS

1 Revise M1 statics and moments, particularly friction. What does limiting equilibrium mean? When is it true that $F = \mu R$? When it isn't true, what can we say about the value of the frictional force?

2 Make a list of all common modelling assumptions in statics. In each case, note down the mathematical consequence.

Example: surface is smooth $\Rightarrow$ *frictional forces can be ignored*

3 Investigate whether or not it is possible for a ladder to rest in equilibrium against a rough wall with its foot on a smooth floor.

CURRENT WORK – MECHANICS PROJECTILES

4 Nick hits a golf ball with initial velocity 50 ms^{-1} at 35º to the horizontal. Calculate:

(*a*) the maximum height that the ball reaches;
(*b*) the horizontal distance it travels before bouncing.

5 Clare scoops a hockey ball off the ground, giving it an initial velocity of 19 ms^{-1} at 25º to the horizontal.

(*a*) Find the time that elapses before the ball hits the ground.
(*b*) Find the horizontal distance the ball travels before hitting the ground.
(*c*) Find how long it takes for the ball to reach its maximum height.
(*d*) Find the maximum height reached.

A member of the opposing team is standing 20 m away from Clare in the direction of the ball's flight.

(*e*) How high is the ball when it passes her? Can she stop the ball?

6 An aircraft is flying at a speed of 300 ms^{-1} and maintaining an altitude of 10 000 m when a bolt becomes detached. Ignoring air resistance, find:

(*a*) the time the bolt takes to reach the ground;
(*b*) the horizontal distance it has travelled in this time;
(*c*) the speed of the bolt when it hits the ground; and
(*d*) the angle to the horizontal at which it hits the ground.

7 Davina Dare-cat, a stunt motorcycle rider, attempts to jump over a gorge 80 m wide. She uses a ramp at 25º to the horizontal for her take-off and has a speed of 30 ms^{-1}at this time.

(*a*) Does Davina cross the gorge successfully?

A different stunt rider, Mini, believes that the effect of air resistance is to reduce her distance by 40%.

(*b*) Under this assumption, calculate her minimum take-off speed for the same jump.

CONSOLIDATION – FP2

8. a) Find the Taylor series for $x^3 \ln x$ in ascending powers of $(x-1)$ up to and including the term in $(x-1)^4$.

   b) Using your series in **a**, find an approximation for $\ln 1.5$, giving your answer to 4 decimal places.

9. Find the Taylor expansion of $\sin 2x$ in ascending powers of $\left(x-\frac{\pi}{6}\right)$ up to and including the term in $\left(x-\frac{\pi}{6}\right)^4$.

10. Use the substitution $y=\frac{z}{x^2}$ to transform the differential equation

$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2x(x+2)\frac{\mathrm{d}y}{\mathrm{d}x}+2(x+1)^2y=\mathrm{e}^{-x}$ into the equation $\frac{\mathrm{d}^2z}{\mathrm{d}x^2}+2\frac{\mathrm{d}z}{\mathrm{d}x}+2z=\mathrm{e}^{-x}$.

Hence solve the equation $x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2x(x+2)\frac{\mathrm{d}y}{\mathrm{d}x}+2(x+1)^2y=\mathrm{e}^{-x}$, giving *y* in terms of *x*.

11. Use the substitution $z=\sin x$ to transform the differential equation

$\cos x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\sin x\frac{\mathrm{d}y}{\mathrm{d}x}-2y\cos^3 x=2\cos^5 x$ into the equation $\frac{\mathrm{d}^2y}{\mathrm{d}z^2}-2y=2(1-z^2)$.

Hence solve the equation $\cos x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\sin x\frac{\mathrm{d}y}{\mathrm{d}x}-2y\cos^3 x=2\cos^5 x$, giving *y* in terms of *x*.

12. Use applications of de Moivre's theorem to prove the following trigonometric identities:

   a) $\sin 5\theta=16\sin^5\theta-20\sin^3\theta+5\sin\theta$

   b) $\sin^5\theta=\frac{1}{16}(\sin 5\theta-5\sin 3\theta+10\sin\theta)$

13. Solve the following equation, expressing your answer for *z* in the form $x+\mathrm{i}y$, where $x\in\mathbb{R}$ and $y\in\mathbb{R}$.

$z^3-\mathrm{i}=0$

14. Solve the following equation, expressing your answer for *z* in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\le\pi$. Give θ to 2d.p.

$z^3=\sqrt{11}-4\mathrm{i}$

CHALLENGE QUESTION

(i) Find the coefficient of x^6 in $\left(1-2x+3x^2-4x^3+5x^4\right)^3$.
You should set our your working clearly.

(ii) By considering the binominal expansions of $(1+x)^{-2}$ and $(1+x)^{-6}$, or otherwise, find the coefficient of x^6 in

$$\left(1-2x+3x^2-4x^3+5x^4-6x^5+7x^6\right)^3$$

Answers:

(4a) 42 m (2sf) (4b) 240 m (2sf)
(5a) 1.6 s (2sf) (5b) 28 m (2sf) (5c) 0.82 s (2sf) (5d) 3.3 m (2sf)
(5e) 2.7 m (2sf); reaching with her hockey stick, quite possibly
(6a) 45 s (2sf) (6b) 14 km (6c) 530 ms–1 (2sf) (6d) 56º
(7a) [70.3510…] No, her range is 70 m (2sf). (7b) 41 ms–1 (2sf)

(8b) 0.4059 (4 d.p.)

(9) $\frac{\sqrt{3}}{2}+1\left(x-\frac{\pi}{6}\right)-\sqrt{3}\left(x-\frac{\pi}{6}\right)^2-\frac{2}{3}\left(x-\frac{\pi}{6}\right)^3+\frac{\sqrt{3}}{3}\left(x-\frac{\pi}{6}\right)^4+\ldots$

(10) $y=\frac{e^{-x}}{x^2}(A\cos x+B\sin x+1)$

(11) $y=Ae^{\sqrt{2}\sin x}+Be^{-\sqrt{2}\sin x}+\sin^2 x$

(12a) $\sin 5\theta=16\sin^5\theta-20\sin^3\theta+5\sin\theta$

(12b) $\sin^5\theta=\frac{1}{16}(\sin 5\theta-5\sin 3\theta+10\sin\theta)$

(13) $z=\frac{\sqrt{3}}{2}+\frac{1}{2}i,-\frac{\sqrt{3}}{2}+\frac{1}{2}i,-i$

(14) $z=\sqrt{3}e^{-0.29i},\sqrt{3}e^{1.80i},\sqrt{3}e^{-2.39i}$

Challenge Question: Eta

Let

$$f(x)=|x+1|-|x|+3|x-1|-2|x-2|-(x+2).$$

We have to solve $f(x)=0$ in the five regions of the x-axis determined by the modulus functions, namely

$$x\leqslant -1;\quad -1\leqslant x\leqslant 0;\quad 0\leqslant x\leqslant 1;\quad 1\leqslant x\leqslant 2;\quad 2\leqslant x.$$

In the separate regions, we have

$$f(x)=\begin{cases}(x+1)-x+3(x-1)-2(x-2)-(x+2) & = & 0 & \text{for} & 2\leqslant x<\infty\\ (x+1)-x+3(x-1)+2(x-2)-(x+2) & = & 4x-8 & \text{for} & 1\leqslant x\leqslant 2\\ (x+1)-x-3(x-1)+2(x-2)-(x+2) & = & -2x-2 & \text{for} & 0\leqslant x\leqslant 1\\ (x+1)+x-3(x-1)+2(x-2)-(x+2) & = & -2 & \text{for} & -1\leqslant x\leqslant 0\\ -(x+1)+x-3(x-1)+2(x-2)-(x+2) & = & -2x-4 & \text{for} & -\infty<x\leqslant -1\end{cases}$$

Solving in each region gives:

(i) $x\geqslant 2$

Here, $f(x)=0$ so the equation is satisfied for all values of x. $f(-2)=0$ and $f(x)=0$ for any $x\geqslant 2$.

(ii) $1\leqslant x\leqslant 2$

Here, $f(x)=0$ only if $x=2$.

(iii) $0\leqslant x\leqslant 1$

Here, $f(x)=0$ only if $x=-1$. This is not a solution since the point $x=-1$ does not lie in the region $0\leqslant x\leqslant 1$.

(iv) $-1\leqslant x\leqslant 0$

Here, $f(x)=-2$ so there is no solution.

(v) $x\leqslant -1$

Here, $f(x)=0$ only if $x=-2$. This is a solution since the point $x=-2$ does lie in the region $x\leqslant -1$.

The equation $f(x)=0$ is therefore satisfied by $x=-2$ and by any x greater than, or equal to, 2.

BHASVIC

MαTHS

ASSIGNMENT COVER SHEET lambda

Name -- Maths Teacher --

Question	Done	Backpack	Ready for test	Notes
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