α	β	γ	δ	ε	ζ	η	θ	ι	**κ**	λ	μ	ν	ξ	o	π	ρ	σ	τ	υ	φ	χ	ψ	ω

"Perhaps the greatest paradox of all is that there are paradoxes in mathematics"

J Newman

Further Maths A2 (M2FP2D1) Assignment κ (kappa) A

PREPARATION *Every week you will be required to do some preparation for future lessons, to be advised by your teacher.*

Prepare the following proofs to reproduce in class:

1. a) Simplify $\cos\left(\frac{\pi}{2}-\theta\right)$.

   b) **Hence** show that $(\sin\theta + \mathrm{i}\cos\theta)^6 \equiv -\cos 6\theta + \mathrm{i}\sin 6\theta$

2. Use de Moivre's theorem to

   a) prove that
   $$(\cos\theta - \mathrm{i}\sin\theta)^n \equiv \cos n\theta - \mathrm{i}\sin n\theta$$
   for n any integer

   b) **Verify** that $\cos\frac{p}{q}\theta + \mathrm{i}\sin\frac{p}{q}\theta$ is a possible value of
   $$(\cos\theta + \mathrm{i}\sin\theta)^{\frac{p}{q}}$$, where p and q are integers, $q \neq 0$.

3. a) Use induction to prove that
   $$(\cos\theta + \mathrm{i}\sin\theta)^n \equiv \cos n\theta + \mathrm{i}\sin n\theta$$
   for all positive integers n.

   b) **Deduce** that
   $$(\cos\theta + \mathrm{i}\sin\theta)^m \equiv \cos m\theta + \mathrm{i}\sin m\theta$$
   holds for all negative integers m

CONSOLIDATION

4. Find the solution subject to the given initial conditions for each of the following differential equations:

   a) $25\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 36x = 18$ $\qquad$ $x = 1$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 0.6$ when $t = 0$

   b) $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 2\frac{\mathrm{d}x}{\mathrm{d}t} + 2x = 2t^2$ $\qquad$ $x = 1$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 3$ when $t = 0$

5. Use applications of de Moivre's theorem to prove the following trigonometric identities:

a) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

b) $\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$

6. Solve the following equation, expressing your answer for *z* in the form $x + \mathrm{i}y$, where $x \in \mathbb{R}$ and $y \in \mathbb{R}$.

$$z^4 - 1 = 0$$

7. Solve the following equation, expressing your answer for *z* in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \le \pi$. Give θ to 2d.p.

$$z^4 = 3 + 4\mathrm{i}$$

8. a) Find the Taylor series expansion of $\sqrt{x}$ in ascending powers of $(x-1)$ as far as the term in $(x-1)^4$.

b) Use your answers in **a** to obtain an estimate for $\sqrt{1.2}$, giving your answer to 3 decimal places.

9. a) Use Taylor's expansion to express each of the following as a series in ascending powers of *x* as far as the term in x^4.

i) $\cos\left(x + \frac{\pi}{4}\right)$ ii) $\ln(x+5)$ iii) $\sin\left(x - \frac{\pi}{3}\right)$

b) Use your result in **ii** to find an approximation for $\ln 5.2$, giving your answer to 4 significant figures.

10. Sketch the graphs of

a) $y = \dfrac{x-1}{x+2}$

b) $y = \left|\dfrac{x-1}{x+2}\right|$

on separate diagrams.

Using your graph of $y = \left|\dfrac{x-1}{x+2}\right|$, find the set of values of *x* for which $\left|\dfrac{x-1}{x+2}\right| < 2$.

11. Shade on an Argand diagram the region represented by:

a) $|z-3|<2$ b) $|z+2\mathrm{i}|>4$

c) $|z+3-2\mathrm{i}|<5$ d) $|z-5-\mathrm{i}|>2$

e) $2<|z-3-2\mathrm{i}|<3$ f) $1<|z+4+\mathrm{i}|<5$

g) $|z-\mathrm{i}|<|z-1|$ h) $|z-2\mathrm{i}|<|z+3-\mathrm{i}|$

i) $|z|<5|z-4|$ j) $|z|>5|z+6|$

12. If z_1 and z_2 are complex numbers, show geometrically that $|z_1-z_2|\le|z_1|+|z_2|$

13. If z_1 and z_2 are complex numbers, show geometrically that $|z_1-z_2|\ge\big||z_1|-|z_2|\big|$

14. Find the sum of all even numbers between 2 and 200 inclusive, excluding those which are multiples of 3.

CHALLENGE QUESTION

Find all the solutions of the equation $|x+1|-|x|+3|x-1|-2|x-2|=x+2$

Answers:

(4a) $x=\frac{1}{2}\left(\cos\frac{6}{5}t+\sin\frac{6}{5}t+1\right)$ (4b) $x=\mathrm{e}^{t}\sin t+1+2t+t^{2}$ or $x=\mathrm{e}^{t}\sin t+(1+t)^{2}$

(5a) $\sin 3\theta=3\sin\theta-4\sin^{3}\theta$ (5b) $\cos^{4}\theta=\frac{1}{8}(\cos 4\theta+4\cos 2\theta+3)$

(6) $z=1,\mathrm{i},-1,-\mathrm{i}$ (7) $z=5^{\frac{1}{4}}\mathrm{e}^{0.23\mathrm{i}},5^{\frac{1}{4}}\mathrm{e}^{1.80\mathrm{i}},5^{\frac{1}{4}}\mathrm{e}^{-1.34\mathrm{i}},5^{\frac{1}{4}}\mathrm{e}^{-2.91\mathrm{i}}$

(8b) 1.095 (3 d.p.) (9a i) $\frac{\sqrt{2}}{2}\left\{1-x-\frac{1}{2}x^{2}+\frac{1}{6}x^{3}+\frac{1}{24}x^{4}-\ldots\right\}$

(9a ii) $\ln 5+\frac{1}{5}x-\frac{1}{50}x^{2}+\frac{1}{375}x^{3}-\frac{1}{2500}x^{4}+\ldots$

(9a iii) $\sin\left(x-\frac{\pi}{3}\right)=\frac{1}{2}\left\{-\sqrt{3}+x+\frac{\sqrt{3}}{2!}x^{2}-\frac{1}{3!}x^{3}-\frac{\sqrt{3}}{4!}x^{4}+\ldots\right\}$

(9b) 1.64866 (6 s.f.) (14) 6734

Challenge question:

Answers will appear on the next assignment.

BHASVIC

MαTHS

ASSIGNMENT COVER SHEET kappa

Name -- Maths Teacher --

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